Inverting Functions with Exponentials and Logs

Professor Sormani

A supplement for MAT172 using Larson and Hostetler Ed 6

Before doing this assignment, review Section 1.8, particularly Examples 6 and 7 on inverses. Here we extend those techniques using our new knowledge from 3.1-4.

Example: Let $f(x) = e^{3x-1}$. What are the domain and range of f? What is the inverse of f? Be sure to state the domain and range of the inverse as well as giving us the formula.

Solution: Well the domain of f is $(-\infty, \infty)$ because there is an output for every possible input. The range is $(0, \infty)$ because all the outputs are positive.

To get a formula for $f^{-1}(x)$, we write $y = e^{3x-1}$ and solve for x.

$$y = e^{3x-1}$$

$$Ln(y) = Ln(e^{3x-1})$$

$$Ln(y) = 3x - 1$$

$$Ln(y) + 1 = 3x$$

$$(Ln(y) + 1)/3 = x$$

We have solved for x, which is the outpu of f^{-1} when the input is y. So replace x by $f^{-1}(x)$ and replace y by x:

$$f^{-1}(x) = (Ln(x) + 1)/3.$$

Note this is NOT the same as $Ln(x + 1)/3$ or $Ln((x + 1)/3)$.

The domain of f^{-1} is the range of f, which is $(0, \infty)$ and that makes sense because we can take $Ln(x)$ for any positive number x.

The range of f^{-1} is the domain of f, is $(-\infty, \infty)$.

Check:

$$\begin{aligned}
f(f^{-1}(x)) &= f((Ln(x) + 1)/3) \text{ plugging in the formula for } f^{-1}(x) && (1)\\
&= e^{3((Ln(x)+1)/3)-1} \text{ substituting into the formula for f} && (2)\\
&= e^{(Ln(x)+1)-1} \text{ cancelling the threes} && (3)\\
&= e^{Ln(x)} \text{ cancelling the ones} && (4)\\
&= x \text{ because Ln is the inverse of } e^x. && (5)
\end{aligned}$$

Also check:

$$
\begin{aligned}
f^{-1}(f(x)) &= f^{-1}(e^{3x-1}) \text{ plugging in the formula for } f(x) && (6)\\
&= (Ln(e^{3x-1})+1)/3 \text{ substituting into the formula for } f^{-1} && (7)\\
&= ((3x-1)+1)/3 \text{ because Ln and e cancel since they are inverses} && (8)\\
&= (3x)/3 = x && (9)
\end{aligned}
$$

Since $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$ we have verified that we have the correct formula for the inverse.

Problems: (answers at the end for problems 1-5)

1. Find the domain, range and inverse for $f(x) = e^{2x+7}$.
2. Find the domain, range and inverse of $f(x) = Ln(x-5)$. *Warning, the Ln is only defined on positive numbers so one needs* $x - 5 > 0$.
3. Find the domain, range and inverse of $f(x) = 5^{x+2}$.
4. Find the domain, range and inverse of $f(x) = Log_2(8x)$.
5. Find the domain, range and inverse of $f(x) = e^x + 3$. *Be careful with the range. Think about how graphs shift.*
6. Find the domain, range and inverse for $f(x) = 2^{4x-5}$.
7. Find the domain, range and inverse of $f(x) = Ln(x+13)$.
8. Find the domain, range and inverse of $f(x) = 2^{x+5}$.
9. Find the domain, range and inverse of $f(x) = Log_5(25x)$.
10. Find the domain, range and inverse of $f(x) = e^{2x} - 10$.

Answers:

1. The domain of f is $(-\infty, \infty)$ and the range is $(0, \infty)$. The inverse is $f^{-1}(x) = (Ln(x) - 7)/2$ and its domain is $(0, \infty)$ and its range is $(-\infty, \infty)$.
2. The domain of f is $(5, \infty)$ and the range is $(-\infty, \infty)$. The inverse is $f^{-1}(x) = e^x + 5$ and its domain is $(-\infty, \infty)$ and its range is $(5, \infty)$.
3. The domain of f is $(-\infty, \infty)$ and the range is $(0, \infty)$. The inverse is $f^{-1}(x) = Log_5(x) - 2$. The domain of f^{-1} is $(0, \infty)$ and the range is $(-\infty, \infty)$.
4. The domain of f is $(0, \infty)$, the range is $(-\infty, \infty)$, the inverse is $f^{-1}(x) = 2^x/8$ or $= 2^{x-3}$ which is the same thing. The domain of f^{-1} is $(-\infty, \infty)$ and the range is $(0, \infty)$.
5. The domain is $(-\infty, \infty)$, the range is $(3, \infty)$, the inverse is $f^{-1}(x) = Ln(x-3)$. The inverse's domain is $(3, \infty)$ which makes sense because the input for Ln must be positive, and the inverse's range is $(-\infty, \infty)$.