Math 241, Fall 2015 **Renato Ghini Bettiol**

Homework Set 5

Due: Oct 20, 2015 (in class)

1. Consider the Sturm-Liouville problem

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((1+x^2)\frac{\mathrm{d}\phi}{\mathrm{d}x}\right)+\lambda\phi=0, \quad \phi(0)=\phi(1)=0.$$

   Use its Rayleigh quotient to show that all the eigenvalues satisfy $\lambda \geq 0$. Then, use the equation and its boundary conditions to show that in fact $\lambda > 0$.

2. Haberman 5.3.8 (or 5.6.2). [Note that these are the exact same problem...]

3. Haberman 5.3.9

4. Haberman 5.3.10

5. Rewrite each of the following equations in Sturm-Liouville form:
   a) Hermite equation: $\phi'' - 2x\phi' + \lambda\phi = 0$;
   b) Chebyshev equation: $(1-x^2)\phi'' - x\phi' + \lambda\phi = 0$;
   c) Laguerre equation: $x\phi'' + (1-x)\phi' + \lambda\phi = 0$;
   d) Bessel equation: $x^2\phi'' + x\phi' + (x^2-\lambda^2)\phi = 0$;
   e) Legendre equation: $(1-x^2)\phi'' - 2x\phi' + \lambda(\lambda+1)\phi = 0$.

6. Consider the boundary value problem

$$\phi'' + (2-4x)\phi' + \lambda\phi = 0, \quad \phi(0)=\phi(1)=0.$$

   a) Rewrite the above equation in Sturm-Liouville form;
   b) Verify that $\phi(x) = x(1-x)$ is an eigenfunction for this problem, and compute its eigenvalue λ;
   c) Prove that the eigenvalue λ obtained in b) is the smallest eigenvalue of this Sturm-Liouville problem.

7. Haberman 5.5.3

8. Haberman 5.5.4

9. Haberman 5.5.9