Math 241, Fall 2015 **Renato Ghini Bettiol**

Homework Set 4

DUE: OCT 13, 2015 (IN CLASS)

1. Consider the function $f\colon [0,\pi] \to \mathbb{R}$ given by $f(x) = x$.
   a) Use an odd extension to write $f(x)$ as a Fourier Sine Series;
   b) Use an even extension to write $f(x)$ as a Fourier Cosine Series;
   c) Use a periodic extension to write $f(x)$ as a Fourier Series (with both Sines and Cosines), and relate it with your answers to a) and b);
   d) For what values of $x \in \mathbb{R}$ are the above Fourier Series equal to the corresponding extension of $f(x)$ used to compute the series?

   HINT: To find the coefficients of the series in a), b), and c), compute the appropriate integrals using integration by parts. To study pointwise convergence of these series, use Fourier's Theorem.

2. Consider the function $g\colon [-L,L] \to \mathbb{R}$ given by $g(x) = x^2$.
   a) Write $g(x)$ as a Fourier Cosine Series;
   HINT: To find the coefficients of this series, use integration by parts and a substitution to reduce the problem to a computation you did in Exercise 1 a).
   b) Use this series at $x = L$ and Fourier's Theorem to prove the following expression
   $$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$$
   c) CHALLENGE (OPTIONAL): Generalize the above to prove the following expression
   $$\frac{\pi^4}{90} = \sum_{n=1}^{\infty} \frac{1}{n^4} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots$$
   If you are curious about the general case $\sum_{n=1}^{\infty} n^{-2k}$, google "Bernoulli numbers".

3. Verify that the Fourier Cosine Series of $g(x) = x^2$ in Exercise 2 can be differentiated term by term (justify why), and compare the result with Exercise 1 a).

4. Haberman 3.2.1 (a), (b), (c), (d)

5. Haberman 3.2.2 (a), (b), (c), (d). *Note that 3.2.2 (f) was solved in class.*

6. Haberman 3.3.1 (a), (b), (c), (d)

7. Haberman 3.4.12

8. Haberman 3.5.1