GRAD STUDENT GEOMETRY AND TOPOLOGY SEMINAR
FALL 2017 PROGRAM

Meeting coordinates: Wednesdays, 12.00pm-1:00pm at DRL 4C4

Long-term topics that we elected to cover, in this order (though possibly only one might be feasible this semester, with the others being carried over to the spring):

(1) **Metric Riemannian Geometry.** The plan is to use [Fuk06] as the main reference, complementing it with the references therein for each of the subtopics, which include Gromov-Hausdorff convergence, finiteness theorems, almost flat manifolds, collapsing manifolds, Morse theory of distance function, Soul Theorem, Alexandrov spaces, and Cheeger-Colding Theory.
(2) **Special holonomy and G_2-manifolds.** Closed manifolds with special holonomy, in particular G_2-manifolds, have attracted considerable interest lately. Possible starting references are the books of Joyce [Joy00, Joy07].
(3) **Total curvature of knotted submanifolds and Morse Theory.** Topics stemming from the Fary-Milnor Theorem and the Chern-Lashof Theorem, see [CL57, Mil50, F́49, Sun76].

Short-term topics we elected to cover this semester, not necessarily in this order:

(1) **Focal radii of orbits** [GS]**.** Every effective action of a compact Lie group on a unit sphere admits an explicit orbit whose principal curvatures are bounded from above by $4\sqrt{14}$.
(2) **4-manifolds with** $\mathrm{Ric} > 0$ **and T^2-action.** The connected sum of any number of copies of $\mathbb{C}P^2$, $\overline{\mathbb{C}P}^2$, and $S^2 \times S^2$ is known to carry metrics with $\mathrm{Ric} > 0$ and to admit isometric T^2-actions. Both of these geometric properties can be realized simultaneously [BkM07].
(3) **Isometry-invariant geodesics.** Existence of geodesics invariant under isometries is studied with Morse theoretic methods [Gro74].
(4) **Besse $\mathbb{R}P^n$'s are standard.** Any metric on $\mathbb{R}P^n$, $n \neq 3$, with all geodesics closed must have constant sectional curvature [LS17].
(5) **Curvature homogeneous metrics on $\mathsf{SL}(2,\mathbb{R})$.** An infinite-dimensional family of complete curvature homogeneous metrics on $\mathsf{SL}(2,\mathbb{R})$ is constructed, using its Iwasawa decomposition and a Berger-type deformation [SW15].
(6) **Gauss map for surface in 4-space.** Topological restrictions are placed on the Gauss map for surfaces in $\mathbb{R}^4$, and the prescribed Gauss map problem is studied [Wei84].
(7) **Minimal Surfaces and Spectral Geometry.** Topics related to minimal surfaces and connections with spectral geometry, following [SY94] and references therein.
(8) **Area bounds for free boundary minimal surfaces.** Sharp area bounds for free boundary minimal surfaces in the unit ball are obtained in [Bre12, BH17].

References

[BH17] S. Brendle and P.-K. Hung. *Area bounds for minimal surfaces that pass through a prescribed point in a ball*. Geom. Funct. Anal., 27 (2017), 235–239.

[BkM07] Y. V. Bazaĭkin and I. V. Matvienko. *On four-dimensional T^2-manifolds of positive Ricci curvature*. Sibirsk. Mat. Zh., 48 (2007), 973–979.

[Bre12] S. Brendle. *A sharp bound for the area of minimal surfaces in the unit ball*. Geom. Funct. Anal., 22 (2012), 621–626.

[CL57] S.-s. Chern and R. K. Lashof. *On the total curvature of immersed manifolds*. Amer. J. Math., 79 (1957), 306–318.

[FÁ49] I. Fáry. *Sur la courbure totale d'une courbe gauche faisant un nœud*. Bull. Soc. Math. France, 77 (1949), 128–138.

[Fuk06] K. Fukaya. *Metric Riemannian geometry*. In *Handbook of differential geometry. Vol. II*, 189–313. Elsevier/North-Holland, Amsterdam, 2006. `https://www.math.kyoto-u.ac.jp/preprint/2004/16fukaya.pdf`.

[Gro74] K. Grove. *Isometry-invariant geodesics*. Topology, 13 (1974), 281–292.

[GS] C. Gorodski and A. B. Saturnino. *Focal radii of orbits*. arXiv:1708.09078.

[Joy00] D. D. Joyce. *Compact manifolds with special holonomy*. Oxford Mathematical Monographs. Oxford University Press, Oxford, 2000.

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[LS17] S. Lin and B. Schmidt. *Real projective spaces with all geodesics closed*. Geom. Funct. Anal., 27 (2017), 631–636.

[Mil50] J. W. Milnor. *On the total curvature of knots*. Ann. of Math. (2), 52 (1950), 248–257.

[Sun76] D. Sunday. *The total curvature of knotted spheres*. Bull. Amer. Math. Soc., 82 (1976), 140–142.

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[Wei84] J. L. Weiner. *The Gauss map for surfaces in* 4*-space*. Math. Ann., 269 (1984), 541–560.