Homework Set 1

1. $(x-1)^2 + (y-2)^2 + (z-3)^2 = 4^2$

$(0,0,0)$: $(-1)^2 + (-2)^2 + (-3)^2 \overset{?}{=} 16$

$14 < 16$

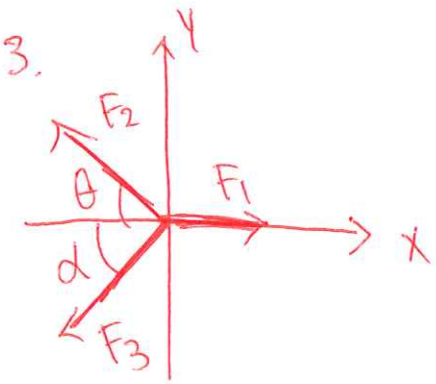
$\therefore$ Origin is inside

2. $\vec{a} = \langle 1, 2, 1 \rangle \quad |\vec{a}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

$\vec{b} = \langle 2, -6, -8 \rangle \quad |\vec{b}| = \sqrt{2^2 + (-6)^2 + (-8)^2} = \sqrt{4+36+64} = \sqrt{104}$

$\vec{c} = \langle -2, 2, 4 \rangle \quad |\vec{c}| = \sqrt{(-2)^2 + 2^2 + 4^2} = \sqrt{24}$

3.



$|F_1| = |F_2| = |F_3| = 1$ (Normalize to 1)

$\vec{F_1} = \langle 1, 0 \rangle$

$\vec{F_2} = \langle -\cos\theta, \sin\theta \rangle \qquad \theta, \alpha \in [0, \frac{\pi}{2}]$

$\vec{F_3} = \langle -\cos\alpha, -\sin\alpha \rangle$

$1 - \cos\theta - \cos\alpha = 0$

$\sin\theta - \sin\alpha = 0$

$\Rightarrow$ $\sin\theta = \sin\alpha$

$\theta = \alpha$; for $[0, \frac{\pi}{2}]$

$1 - 2\cos\theta = 0$

$\theta = \alpha = \frac{\pi}{3}$

$\therefore \vec{F_1} = \langle 1, 0 \rangle$

$\vec{F_2} = \langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$

$\vec{F_3} = \langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$

4. $\vec{V_1} = \langle x, -x, 1 \rangle$

$\vec{V_2} = \langle 2, x, -1 \rangle$

$\vec{V_1} \cdot \vec{V_2} = 0 = 2x - x^2 - 1$

$0 = x^2 - 2x + 1$

$\boxed{x = 1}$

$|V_1| = 1 = \sqrt{x^2 + (-x)^2 + 1}$

$1 = \sqrt{2x^2 + 1}$

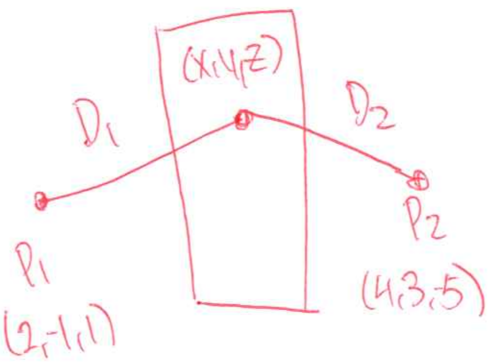
$\boxed{x = 0}$

$|V_2| = 1 = \sqrt{2^2 + x^2 + (-1)^2}$

$1 = \sqrt{x^2 + 5}$

No solution

5. Method #1



$D_1 = D_2$

$\sqrt{(x-2)^2 + (y+1)^2 + (z-1)^2} = \sqrt{(x-4)^2 + (y-3)^2 + (z+5)^2}$

$x^2 - 4x + 4 + y^2 + 2y + 1 + z^2 - 2z + 1 =$

$x^2 - 8x + 16 + y^2 - 6y + 9 + z^2 + 10z + 25$

$-4x + 2y - 2z + 6 = -8x - 6y + 10z + 50$

$\boxed{4x + 8y - 12z - 44 = 0}$ * or any scalar multiple

Method #2

$\vec{n} = \vec{P_1P_2} = \langle 2, 4, -6 \rangle$

$\frac{P_1 + P_2}{2} = (3, 1, -2)$

$Ax + By + Cz + D = 0$

$\vec{n} = \langle A, B, C \rangle$

$2x + 4y - 6z + D = 0$

$2(3) + 4(1) - 6(-2) + D = 0$

$6 + 4 + 12 + D = 0$

$D = -22$

$\boxed{2x + 4y - 6z - 22 = 0}$

or $x + 2y - 3z - 11 = 0$