Homework Set 9

DUE: APR 6, 2017 (IN CLASS)

1. Compute the Jacobian of the following transformations:

    a) Polar coordinates: $G\colon \mathbb{R}^2 \to \mathbb{R}^2$, $G(r,\theta) = (r\cos\theta, r\sin\theta)$
    b) Cylindrical coordinates: $G\colon \mathbb{R}^3 \to \mathbb{R}^3$, $G(r,\theta,z) = (r\cos\theta, r\sin\theta, z)$
    c) Spherical coordinates: $G\colon \mathbb{R}^3 \to \mathbb{R}^3$, $G(r,\theta,\phi) = (r\cos\theta\sin\phi, r\sin\theta\sin\phi, r\cos\phi)$
    d) $G\colon \mathbb{R}^3 \to \mathbb{R}^3$, $G(u,v,w) = (au+1, bv-2, cw+3)$, where a, b, c are constants.
    e) $G\colon \mathbb{R}^2 \to \mathbb{R}^2$, $G(u,v) = \left(\frac{v-u}{2}, \frac{u+v}{2}\right)$

2. Find the volume of the solid $\Omega \subset \mathbb{R}^3$ bounded by the surface given in spherical coordinates by the equation $r = (\sin\phi)^{1/3}$.

3. Find the volume inside the cylinder $x^2 + y^2 = 1$, below the plane $x + y + z = 2$, above the xy-plane and in the first octant.

4. Let Ω be the triangle with vertices at the origin, $(2,0)$ and $(0,2)$. Use the change of variables $u = y - x$, $v = y + x$, to evaluate the integral $\displaystyle\iint_D e^{\frac{y-x}{y+x}}\,\mathrm{d}A$

5. Compute the following line integrals of functions and vector fields:

    a) $\int_\gamma f(x,y,z)\,\mathrm{d}s$, where $\gamma(t) = (t, 1-t, 0)$, $0 \le t \le 1$, and $f(x,y,z) = 2x + 3y$
    b) $\int_\gamma f(x,y,z)\,\mathrm{d}s$, where $\gamma(t) = (\cos t, \sin t, 2t)$, $0 \le t \le 2\pi$, and $f(x,y,z) = \sqrt{x^2+y^2}$
    c) $\int_\gamma \vec{F}\,\mathrm{d}\gamma$, where $\gamma(t) = (t^2, t, -t)$, $0 \le t \le 1$, and $\vec{F}(x,y,z) = (xy, -x, yz)$
    d) $\int_\gamma \vec{F}\,\mathrm{d}\gamma$, where $\gamma(t) = (\sin t, \cos t, t)$, $0 \le t \le \pi$, and $\vec{F}(x,y,z) = (3z, y^2, 4x)$