

## Homework Set 6

DUE: MAR 2, 2017 (IN CLASS)

1. Suppose a lizard is sitting at the point  $(1, 3)$  of a heated plane whose temperature at the point  $(x, y)$  is given by  $T(x, y) = x^2 + y^2 - x - y$ . In what direction should the lizard move to increase the temperature as fast as possible?
2. Find the tangent plane to the following surfaces at the following points:
  - (a)  $x + y - z = 3$  at  $(1, 1, -1)$
  - (b)  $x^2 + y^2 + z^2 = 4$  at  $(1, -1, \sqrt{2})$
  - (c)  $x^2 + 2xy - y^2 + z^2 = 5$  at  $(1, 0, 2)$
  - (d)  $e^{xz} + yz = x + z + 1$  at  $(0, 1, 2)$

3. Consider the following smooth functions:

$$f(x, y) = x^2 - y^2, \quad g(x, y) = x^4 + y^4 + 4xy, \quad h(x, y, z) = \frac{1}{x} + xy + \frac{1}{y} + z^2$$

For each of them, do the following:

- (a) Compute its gradient
  - (b) Find its critical points
  - (c) Decide (whenever possible) if they are local minima, local maxima, or saddles.
4. Find the absolute minimum and absolute maximum of  $f(x, y) = x^2 - xy + y^2$  in the square  $0 \leq x \leq 1, 0 \leq y \leq 1$ .
  5. What are the points of the ellipse  $x^2 + xy + y^2 = 1$  that are nearest and farthest from the origin?