Math 114, Spring 2017 **Renato Ghini Bettiol**

Homework Set 11

Due: Apr 20, 2017 (in class)

1. Let $f(x,y,z) = x^2 + y^2 + z^2$ and $g(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$ for all $(x,y,z) \neq 0$. Compute:

   a) $\Delta f = \operatorname{div} \nabla f$

   b) $\Delta g = \operatorname{div} \nabla g$

   c) $\nabla f \cdot \nabla g$

   d) $\nabla f \times \nabla g$

2. Parametrize the ellipsoid $(\frac{x}{a})^2 + (\frac{y}{b})^2 + (\frac{z}{c})^2 = 1$, where $a, b, c > 0$ and write down (but do not evaluate) an integral formula for its area.

3. Compute the following surface integrals of real-valued functions:

   a) $\iint_\Sigma y \, \mathrm{d}\Sigma$ where Σ is the surface $z = x + y^2$, $0 \leq x \leq 1$, $0 \leq y \leq 2$.

   b) $\iint_\Sigma xz \, \mathrm{d}\Sigma$ where Σ is the triangle with vertices $(1,0,0)$, $(1,1,1)$, and $(0,0,2)$.

   c) $\iint_\Sigma z(x^2+y^2) \, \mathrm{d}\Sigma$ where Σ is the upper hemisphere $x^2+y^2+z^2 = 4$, $z \geq 0$.

4. Compute the following surface integrals of vector fields:

   a) $\iint_\Sigma \vec{F} \, \mathrm{d}\Sigma$ where $\vec{F}(x,y,z) = (x,y,z)$ and Σ is the sphere $x^2+y^2+z^2 = 9$.

   b) $\iint_\Sigma \vec{F} \, \mathrm{d}\Sigma$ where $\vec{F}(x,y,z) = (x+y)\vec{\imath} - (2y+1)\vec{\jmath} + z\vec{k}$ and Σ is the rectangle with vertices $(1,0,0)$, $(1,0,1)$, $(0,1,0)$, and $(0,1,1)$, oriented such that the outward unit normal $\vec{n}$ satisfies $\vec{n} \cdot \vec{\jmath} > 0$.

   c) $\iint_\Sigma \vec{F} \, \mathrm{d}\Sigma$ where $\vec{F}(x,y,z) = (-x,-y,z^2)$ and Σ is the portion of the cone $z = \sqrt{x^2+y^2}$ between the planes $z = 1$ and $z = 2$, oriented such that the outward unit normal $\vec{n}$ satisfies $\vec{n} \cdot \vec{k} < 0$.

5. Use Stokes' Theorem to compute the following integrals:

   a) $\iint_\Sigma \vec{F} \, \mathrm{d}\Sigma$ where $\vec{F}(x,y,z) = (0,0,x)$ and Σ is the surface $z = x(1-x)y(1-y)$, with $0 \leq x \leq 1$, $0 \leq y \leq 1$. Hint: Note that $\vec{F} = \operatorname{curl} \vec{G}$ where $\vec{G}(x,y,z) = \big(0, \frac{1}{2}x^2, 0\big)$.

   b) $\int_\gamma \vec{F} \, \mathrm{d}\gamma$ where $\vec{F}(x,y,z) = (z^2, y^2, x)$ and γ is the boundary of the triangle with vertices $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$ with counterclockwise orientation.

   c) $\iint_\Sigma \vec{F} \, \mathrm{d}\Sigma$ where $\vec{F}(x,y,z) = x^3 e^y \vec{\imath} - 3x^2 e^y \vec{\jmath}$ and Σ is the unit upper hemisphere $x^2+y^2+z^2 = 1$, $z \geq 0$, with outward pointing unit normal.