Math 110, Spring 2016
HWK12 due Apr 22

1. Let $\mathbf{v}$ be the vector $-3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$. Compute a unit vector parallel to $\mathbf{v}$.

2. The vector $\mathbf{v} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ has magnitude 10. Its direction is 30° counterclockwise from pointing directly east. What are a and b?

3. Let $\mathbf{v}$ be the vector $14\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$, let $\mathbf{w}$ be the vector of length $5\sqrt{2}$ in the "Northeast" diagonal direction, and let $\mathbf{u}$ be the unit vector pointing in a direction fifteen degrees below the $\hat{\mathbf{i}}$ direction.

   (a) Draw these three vectors.

   (b) Compute $\mathbf{w} \cdot \mathbf{u}$.

   (c) Compute $\mathbf{v} \cdot \mathbf{w}$.

4. Let $f(x, y) = \dfrac{1}{6x + y^2}$.

(a) Compute $\nabla f(x, y)$ and evaluate it at the point $(-2, 3)$.

(b) Compute the rate at which $f(x, y)$ changes if (x, y) starts at $(-2, 3)$ and moves at unit speed in the positive x direction.

(c) Compute the rate at which $f(x, y)$ changes at time $t = 0$ if (x, y) traces out the curve $x(t) = -2 + t, y(t) = 3 - t/2$.

(d) In which direction should you move from $(-2, 3)$ in order for f to increase the fastest?

(e) How fast does f increase per unit moved in this direction?

5. The temperature is given by $T(x, y) = e^x\sqrt{1+y}$. A bug starts at the point $(0, 3)$. Which direction should the bug walk in order to increase its temperature the fastest? Your answer should be a unit vector.

6. Suppose that a new graduate's job satisfaction is modeled as a function $u(x, y)$ where x is the annual salary and y is the amount of hours worked per year. Our model assumes [1] that

$$u(x, y) = (4200 - y)\, x^{0.6}\,.$$

   (a) Compute ∇u.

   (b) Evaluate $\nabla u(x, y)$ at the point $x = \$100,000$ and $y = 4000$. Please simplify fractional powers when there is a nice simplification.

   (c) The indifference curve through this point is the set of (x, y) having the same utility function. What is the slope of the indifference curve at the point $(100000, 4000)$ and what does this say about how much you'd have to increase the salary to get this overworked Wharton graduate (4000 hours is an 80 hour work week) to work each extra hour?

[1] The power 0.6 is due to empirical data on the marginal value of a dollar, and the factor of $4000 - y$ is due to the assumption that the most a person could reasonably work on a regular basis is 84 hours (12 hours/day, 7 days/week), therefore $4200 - y$ represents the amount of free time.