Math 110, Spring 2016
HWK11 due Apr 13

1. Let X represent the annual income of a randomly sampled Wharton graduate (measured in hundreds of thousands of dollars) and let Y represent the fraction of this income the person is willing to spend yearly on a car loan. A probability model for the random pair (X, Y) is that it has density Cxe^{-x} on the region $x \geq 0$, $0.2 \geq y \geq 0$.

   (a) What is the normalizing constant, C?

   (b) What is the mean of X?

   (c) What is the mean of Y?

   (d) Set up an integral to compute the probability that a randomly sampled Wharton graduate is willing to pay more than \$20,000 annually for their car.

   (e) Can you do the integral?

2. Let $f(x, y) = \sqrt{x^2 + xy + y^2}$.

   (a) Compute $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at the point $(3, 5)$.

   (b) Use the partial derivatives to estimate $f(3.1, 5)$.

   (c) Use the partial derivatives to estimate $f(2.9, 5.2)$.

   (d) Use the partial derivatives to evaluate the slope of the tangent line to the curve $f(x, y) = 7$ at the point $(3, 5)$. This will be an exact computation, not an estimate.

3. The number of visitors to a national park depends on the ticket prices and the level of staffing. Let V, T and S denote these variables (in respective units of people per year, dollars and full time employees).

   (a) What is the interpretation of the quantity $\dfrac{\partial V}{\partial T}$?

   (b) Is $\dfrac{\partial V}{\partial T}$ likely to be greater or less than zero?

   (c) What is the interpretation of the quantity $\dfrac{\partial V}{\partial S}$?

   (d) Is $\dfrac{\partial V}{\partial S}$ likely to be greater or less than zero?

   (e) In an effort to make the park self-supporting, Congress has pegged the staffing to ticket prices via the formula $S = 10 + 3T$, where S_0 and k are constants. Explain, in this context, the meaning of the total derivative $\dfrac{dV}{dT}$ and give a formula for this in terms of the partial derivatives of V with respect to T and S.

   (f) Suppose $V(T, S) = 50,000\, e^{-T/10} \dfrac{S}{S + 45}$. Compute $\dfrac{dV}{dT}$ when $T = 15$

4. Suppose that x and y vary along a piece of the curve $x^3 - 3xy + y^3 = 3$ in the first quadrant.

   (a) Differentiate implicitly using x as the independent variable and y as the dependent variable.

   (b) Use this to evaluate dy/dx at the point $(1, 2)$.

   (c) Now differentiate implicitly with y being the independent variable and x the dependent variable and use this to evaluate dx/dy at the point $(1, 2)$.

   (d) Explain the relation between the answers in parts (b) and (c).

5. (Problem 66 from Section 14.4 of the textbook): Find the value of $\partial x/\partial z$ at the point $(1, -1, -3)$ if the equation

$$xz + y \ln x - x^2 + 4 = 0$$

defines x as a function of the two independent variables y and z.