Math 110, Spring 2016
HWK09 due Mar 30

1. (Problem # 38 in Section 7.2): The half-life of Polonium-210 is 139 days. You obtain a small amount of it, and need to use it before it is 95% gone, after which there will no longer be enough to be useful for its intended purpose (guess whatd!). For about how many days will you be able to use the polonium?

2. (Problem # 36 in Section 7.2): To encourage buyers to place 100-unit orders, your firm's sales department applies a continuous discount that makes the unit price a function $p(x)$ of the number x of units ordered. The discount decreases the price at the rate of 1% per unit ordered. The price per unit for a 100-unit order is $p(100) = \$20.09$.

   (a) Find $p(x)$ by solving the following initial value problem:

$$\frac{dp}{dx} = -\frac{1}{100}\,p\ ;\ p(100) = 20.09\,.$$

   (b) Find the unit price $p(10)$ for a 10 unit order and the unit price $p(90)$ for a 90 unit order.

   (c) The sales department has asked you to find out if it is discounting so much that the firm's revenue $r(x) = x \cdot p(x)$ will actually be less for a 100 unit order than, say, for a 90 unit order. Reassure them by showing that the maximum value of $r(x)$ occurs at $x = 100$.

   (d) Graph the revenue function $r(x)$ for $0 \leq x \leq 200$.

3. Solve the differential equation you wrote for Problem #8 on the differential equations word problem worksheet:

> Money is deposited in a bank account with an annual interest rate of 3% compounded continuously. What is the amount in the account at time t after an initial deposit of \$1000, if money is being added to the account continuously at a rate of \$500 per year, and no withdrawals are made?

4. When the interest rate is reasonably small, the relation between the continuous rate r and the annualized rate A can be approximated by a Taylor polynomial. Compute the linear and quadratic Taylor polynomials for each in terms of the other.

   (a) The linear approximation for A as a function of r when r is near zero is given by

   $A(r) \approx P_1(r) =$

   (b) The quadratic approximation for A as a function of r when r is near zero is given by

   $A(r) \approx P_2(r) =$

   (c) The linear approximation for r as a function of A when A is near zero is given by

   $r(A) \approx P_1(A) =$

   (d) The quadratic approximation for r as a function of A when A is near zero is given by

   $r(A) \approx P_2(A) =$

5. Solve these separable equations.

   (a) $u' = t^2\sqrt{u}$

   (b) Problem #7.2.15: $\sqrt{x}\dfrac{dy}{dx} = e^{y+\sqrt{x}}$

6. For each of these separable equations, say whether the solution grows without bound, approaches a finite limit as $x \to \infty$ or blows up ($y \to \infty$ at some finite value of x). If it depends on the initial conditions, please be sure to point this out. You do not have to solve the equation.

(a)

$$\frac{dy}{dx} = \frac{y^2}{1+x}$$

(b)

$$y' - \sqrt{1+y}e^{-x} = 0$$

(c)

$$y' = \frac{x^{3/2}}{1+y^2}$$

(d)

$$2yy' = \sqrt{x+x^5}$$

7. Solve these first order linear differential equations. You may need to introduce notation for the antiderivative of any function you are not able to integrate.

   (a) (Problem # 6 in Section 9.2):

   $$(1 + x)y' + y = \sqrt{x}$$

   (b) $y' - \dfrac{y}{\ln t} = 1$ (you can assume $t > 1$)

8. (Problem # 15 in Section 9.2): Solve this first order linear initial value problem.

$$\frac{dy}{dt} + 2y = 3;\ y(0) = 1\,.$$

9. Solve this first order linear initial value problem. You may need to introduce notation for the antiderivative of any function you are not able to integrate.

$$y' - \frac{y}{\sqrt{1+x^4}} = 0\ ;\ y(0) = 1\,.$$

10. (Problem # 19 in Section 9.2): Solve this first order linear initial value problem.

$$(x+1)\frac{dy}{dx} - 2(x^2+x)y = \frac{e^{x^2}}{x+1}\ ;\ y(0) = 5\,.$$