Math 110, Spring 2016
HWK07 due Mar 16

1. The *alternating harmonic series* is the series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$.

   (a) Write the first five terms of the alternating harmonic series.

   (b) To three decimals, give the first five partial sums $A_M = \displaystyle\sum_{n=1}^{M}(-1)^{n+1}\frac{1}{n}$.

   (c) Does the alternating harmonic series converge? In other words, does $\lim_{M\to\infty} A_M$ exist? Why or why not. If you use a test or Theorem from Section 10.6 of the textbook, please explain how it applies.

   (d) Concerning the numeric value of $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$:

      (i) Give any nontrivial upper bound
      (ii) Give any nontrivial lower bound
      (iii) What is your best guess as to the value?

2. (Problem #22 from page 650 of the textbook) Eugene the Evil TA claims he has found a polynomial that is equal to $\sin x$. How can you prove he is lying?

3. (Problem #23 from page 650 of the textbook) Find the value of L such that

   $\lim_{x \to 0} \dfrac{\sin(Lx) - \sin x - x}{x^3}$ exists and evaluate the limit.

   [Hint: you can do this by seeing which value of L allows you to use L'Hôpital's rule three times, or you can do it by writing a cubic Taylor series for $\sin x$ and $\sin(Lx)$.]

4. (Problem #44 from Section 10.3 of the textbook) Are there values of y for which the series $\sum_{n=1}^{\infty} \frac{1}{ny}$ converges? Find such a vaue of y (please justify your answer) or argue that none exists.

5. Suppose that the number of likes per unit time generated by a fake grassroots social media campaign is well approximated by $C(1+t)^{-1/2}$. The campaign is carried out by web bots that run themselves, so it is possible to let it run for months, years, decades, even centuries. If run to time (essentially) infinity, would we expect the total number of likes to become infinite, or is the accumulated total approaching some finite limit[1]?

[1]Note: probably the bots can't really run for more than a few years maximum without becoming obsolete due to new versions of the social media apps; that's why this math problem is an idealization; still, it conveys a basic sense of whether a rate diminishing as $C(1+t)^{1/2}$ will generate unbounded totals or approach a finite bound.