Math 110, Spring 2016
HWK05 due Feb 24

1. For each of these functions $f(x)$, write a simple function, either cx^p or an exponential ce^{kx}, asymptotically equal to f as $x \to \infty$. You don't need to state a reason. The first one is done for you as an example (we gave a reason but you won't have to).

   (a) $f(x) = \sqrt{e^x - 1}$.

   ANSWER: $f(x) \sim e^{(1/2)x}$. Reason: because 1 is a lot smaller than e^x, we can ignore the 1, so that $\sqrt{e^x - 1} \sim \sqrt{e^x} = e^{x/2}$.

   (b) $f(x) = \dfrac{1}{\sqrt{x - 9}}$

   (c) $f(x) = \dfrac{1}{e^x - x}$

   (d) $f(x) = \dfrac{x}{\sqrt{x^4 - 1}}$

2. For each of these Type I integrals, write it as a limit, then say whether or not the limit converges to a finite value. Again, the first one is done for you. Note: this problem relies on the previous problem.

   (a) $\displaystyle\int_1^\infty \sqrt{e^x - 1}$

   ANSWER: This is equal to $\displaystyle\lim_{M\to\infty}\int_1^M \sqrt{e^x - 1}\,dx$. Because the integrand is $\sim e^{(1/2)x}$ (see previous problem) and $\int_1^\infty e^{kx}$ diverges when $k > 0$, we conclude that the integral is divergent.

   (b) $\displaystyle\int_2^\infty \frac{dx}{\sqrt{x-9}}$

   (c) $\displaystyle\int_1^\infty \frac{dx}{e^x - x}$

   (d) $\displaystyle\int_2^\infty \frac{x\,dx}{\sqrt{x^4 - 1}}$

3. In each case, find a function g of the form $\dfrac{c}{x-a}$ such that $f \sim g$ as $x \to a$.

(a) $f(x) = \dfrac{1}{x^2-1}$; $a = 1$

(b) $f(x) = \dfrac{x+1}{x^2-5x+6}$; $a = 2$

4. Suppose $0 < a < 10$ and $c \neq 0$. Does $\displaystyle\int_0^{10} \frac{c}{x-a}\,dx$ converge or not? Why?

5. The size of an astronomical object is modeled by a random variable X with density $\dfrac{C}{x(\ln x)^2}$ on the interval $[e, \infty]$, measured in kilograms.

(a) What is C?

(b) What is the median of this probability distribution?

(c) What is the 95^{th} percentile of this distribution?

6. The duration in minutes of a medical procedure is modeled by a random variable with probability density $C(1+t)^{-1/2}$ on the interval $[0, 120]$.

   (a) What is C?

   (b) What is the average duration of the treatment?