Math 110, Spring 2016
HWK01 due Feb 3

1. We are going to invest $\$100,000$ in an account that earns interest at a rate of 7.5% for 54 months. Determine how much money will be in the account if:

   (a) interest is compounded quarterly.

   (b) interest is compounded monthly.

   (c) interest is compounded continuously.

2. Define a function f on the interval $(1, \infty)$ by $f(x) = x^{1/\ln x}$. Sketch the graph of f and provide justification as to why your sketch looks the way it does. Please do not use a graphing calculator or WolframAlpha etc. to graph f.

3. Use L'Hôpital's rule to prove these facts about exponentials, logarithms and orders of growth.

   (a) For any integer n, $x^n = o(e^x)$.

   (b) For any integer n, $\ln x \ll x^{1/n}$.

4. In each case, do three things. (1) Write the statement as a limit statement; (2) determine whether the statement is true or false; (3) make a quick sketch of the two functions as they head to the limit, labeled as to which is which, and exhibiting the limiting behavior.

   (a) $\ln(2x) \sim \ln(x)$ as $x \to \infty$

   (b) $e^{2x} \ll x\, e^x$ as $x \to \infty$

(c) $\sqrt{x^2+1} - x \sim \dfrac{1}{2x}$ as $x \to \infty$

(d) $(2-x)^{1/2} = o(2-x)^{1/3}$ as $x \to 2^-$

5. Famous story[1]: In ancient times a Persian king wanted to reward a young man who had saved the life of his daughter. He pointed to the royal chessboard and offered the young man a choice. He could either haave 100 gold coins on the first square, 200 on the second, 300 on the third, and so on, or else have one grain of wheat on the first square, two grains of wheat on the second square, four on the third, eight on the fourth, and so on. Which should the young man have chosen? Please use logs to estimate how many grains of wheat were on the last square or on the whole board. Make any reasonable assumptions about the price of wheat.

[1]For one version, see http://www.dr-mikes-math-games-for-kids.com/rice-and-chessboard.html .

6. (a) Determine whether $2^x \ll x^2$, or vice versa or neither.

(b) Explain the relevance of this to the previous problem.