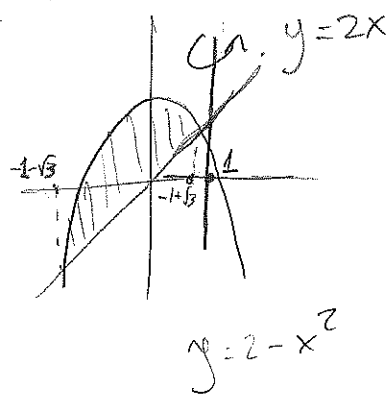


#1



$$2x = 2 - x^2 \Leftrightarrow x = -1 \pm \sqrt{3} < 1$$

Cyl. shell Method;

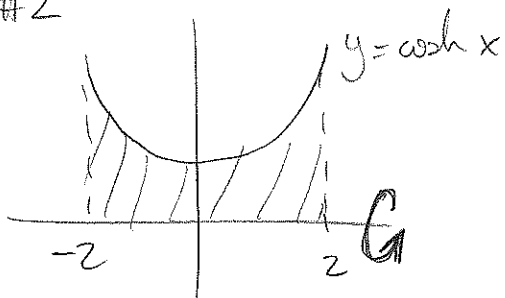
$$V = 2\pi \int_{-1-\sqrt{3}}^{-1+\sqrt{3}} (1-x) \cdot (2-x^2-2x) dx$$

[shell rad] [shell height]

$$= 2\pi \int_{-1-\sqrt{3}}^{-1+\sqrt{3}} x^3 + x^2 - 4x + 2 dx$$

$$= 2\pi \left(\frac{x^4}{4} + \frac{x^3}{3} - 2x^2 + 2x \right) \Big|_{-1-\sqrt{3}}^{-1+\sqrt{3}} = \dots = \boxed{16\pi\sqrt{3}}$$

#2



Disk Method;

$$V = \pi \int_{-2}^2 (\cosh x)^2 dx = \pi \int_{-2}^2 \frac{(e^x + e^{-x})^2}{4} dx$$

$$= \pi \int_{-2}^2 \frac{e^{2x} + 2 + e^{-2x}}{4} dx = \frac{\pi}{4} \frac{e^{2x}}{2} \Big|_{-2}^2 + \frac{\pi}{2} \cdot 4 + \frac{\pi}{4} \frac{e^{-2x}}{-2} \Big|_{-2}^2 =$$

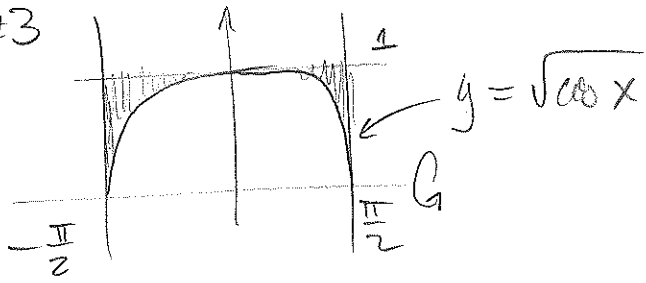
$$= \frac{\pi}{8} (e^4 - e^{-4}) + 2\pi - \frac{\pi}{8} (e^{-4} - e^4) = \boxed{\frac{\pi}{4} (e^4 - e^{-4}) + 2\pi}$$

Note: Alternatively, note $\cosh^2 x = \frac{1 + \cosh(2x)}{2}$ (prove it!) hence:

$$V = \pi \int_{-2}^2 \cosh^2 x dx = \pi \int_{-2}^2 \frac{1 + \cosh(2x)}{2} dx = \frac{\pi}{2} \cdot 4 + \frac{\pi \sinh 2x}{4} \Big|_{-2}^2 = \boxed{2\pi + \frac{\pi \sinh(4)}{2}}$$

sinh odd, 1

#3



Washer Method

$$V = \pi \int_{-\pi/2}^{\pi/2} (1 - \cos x) dx$$

[out]² - [in]²

$$= \pi \left(\pi - \sin x \Big|_{-\pi/2}^{\pi/2} \right) = \pi^2 - \pi(1 - (-1)) = \boxed{\pi^2 - 2\pi}$$

$$\#4 \quad y = \frac{x^4}{4} + \frac{1}{8x^2}, \quad 1 \leq x \leq \sqrt{2}$$

$$y' = x^3 - \frac{1}{4x^3}$$

$$1 + (y')^2 = 1 + x^6 - \frac{1}{2} + \frac{1}{16x^6}$$

$$= x^6 + \frac{1}{2} + \frac{1}{16x^6}$$

$$= \left(x^3 + \frac{1}{4x^3} \right)^2$$

$$\text{Arc length} = \int_1^{\sqrt{2}} \left(x^3 + \frac{1}{4x^3} \right) dx = \left(\frac{x^4}{4} - \frac{1}{8x^2} \right) \Big|_1^{\sqrt{2}} = \left(\frac{4}{4} - \frac{1}{16} \right) - \left(\frac{1}{4} - \frac{1}{8} \right)$$

$$= \frac{15}{16} - \frac{1}{8} = \boxed{\frac{13}{16}}$$

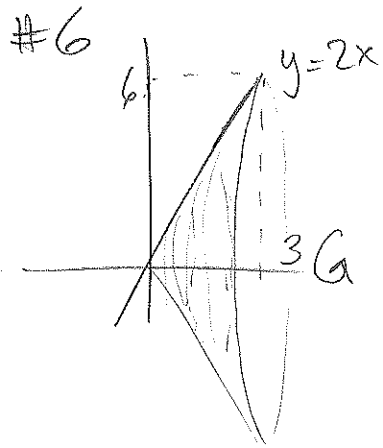
$$\#5 \quad y = \frac{e^x}{4} + \frac{1}{e^x}, \quad 0 \leq x \leq 1$$

$$y' = \frac{e^x}{4} - \frac{1}{e^x}, \quad 1 + (y')^2 = 1 + \frac{e^{2x}}{16} - \frac{1}{2} + \frac{1}{e^{2x}} = \frac{e^{2x}}{16} + \frac{1}{2} + \frac{1}{e^{2x}}$$

$$= \left(\frac{e^x}{4} + \frac{1}{e^x} \right)^2$$

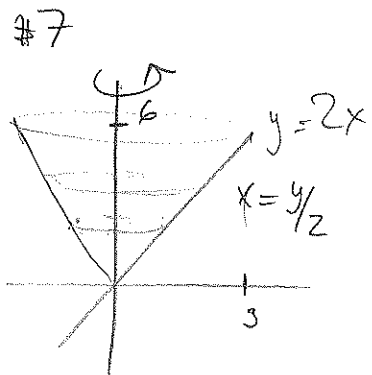
$$\text{Arc length} = \int_0^1 \frac{e^x}{4} + \frac{1}{e^x} dx = \left(\frac{e^x}{4} - \frac{1}{e^x} \right) \Big|_0^1 = \left(\frac{e}{4} - \frac{1}{e} \right) - \left(\frac{1}{4} - 1 \right)$$

$$= \boxed{\frac{e}{4} - \frac{1}{e} + \frac{3}{4}}$$



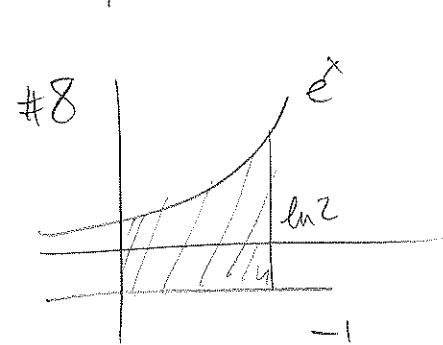
Surface Area

$$S = 2\pi \int_0^3 2x \sqrt{1+4} dx = 4\sqrt{5} \pi \frac{x^2}{2} \Big|_0^3 = \boxed{18\sqrt{5}\pi}$$



Surface Area

$$S = 2\pi \int_0^6 \frac{y}{2} \sqrt{1+\frac{1}{4}} dy = \frac{\pi\sqrt{5}}{2} \frac{y^2}{2} \Big|_0^6 = \boxed{9\sqrt{5}\pi}$$



$$\bar{x} = \frac{\int_0^{\ln 2} (e^x + 1)x dx}{\int_0^{\ln 2} e^x + 1 dx}, \quad \bar{y} = \frac{\frac{1}{2} \int_0^{\ln 2} e^{2x} - 1 dx}{\int_0^{\ln 2} e^x + 1 dx}$$

$$\int_0^{\ln 2} e^x + 1 dx = (e^x + x) \Big|_0^{\ln 2} = 2 + \ln 2 - 1 = 1 + \ln 2$$

$$\int_0^{\ln 2} x e^x + x dx = x e^x \Big|_0^{\ln 2} - \int_0^{\ln 2} e^x dx + \frac{x^2}{2} \Big|_0^{\ln 2} = 2 \ln 2 - 2 + 1 + \frac{(\ln 2)^2}{2}$$

$$= \frac{(\ln 2)^2}{2} + \ln 4 - 1$$

$$\int_0^{\ln 2} e^{2x} - 1 \, dx = \left(\frac{e^{2x}}{2} - x \right) \Big|_0^{\ln 2} = \left(\frac{e^{2\ln 2}}{2} - \ln 2 \right) - \frac{1}{2} = 2 - \ln 2 - \frac{1}{2}$$

$$= \frac{3}{2} - \ln 2.$$

So

$$\bar{x} = \frac{\frac{(\ln 2)^2}{2} + \ln 4 - 1}{1 + \ln 2}, \quad \bar{y} = \frac{\frac{3}{4} - \frac{\ln 2}{2}}{1 + \ln 2}$$

#9

a) $\int \sin x \cos^3 x \, dx = -\frac{\cos^4 x}{4} + C$

$$\int_0^{\pi/2} \sin x \cos^3 x \, dx = -\frac{\cos^4 x}{4} \Big|_0^{\pi/2} = 0 + \frac{1}{4} = \frac{1}{4}$$

b) $\int \sin^2 x \cos^3 x \, dx = \int \sin^2 x (1 - \sin^2 x) \cos x \, dx$

$$\left. \begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array} \right\} \int u^2 (1 - u^2) \, du = \int u^2 - u^4 \, du = \frac{u^3}{3} - \frac{u^5}{5} + C$$

$$= \frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$$

$$\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx = \left(\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x \right) \Big|_0^{\pi/2} = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$$

c) $\int \sin^3 x \cos^3 x \, dx = \int (1 - \cos^2 x) \sin x \cos^3 x \, dx =$

$$\left. \begin{array}{l} u = \cos x \\ du = -\sin x \, dx \end{array} \right\} \int (1 - u^2) u^3 \, du = -\int u^3 - u^5 \, du =$$

$$= -\frac{u^4}{4} + \frac{u^6}{6} + C = -\frac{1}{4} \cos^4 x + \frac{1}{6} \cos^6 x + C$$

$$\int_0^{3\pi/2} \sin^3 x \cos^3 x dx = \left(-\frac{1}{4} \cos^4 x + \frac{1}{6} \cos^6 x \right) \Big|_0^{3\pi/2} = - \left(-\frac{1}{4} + \frac{1}{6} \right) = \boxed{\frac{1}{12}}$$

$$d) \int \tan^4 x \sec^4 x dx = \int \tan^4 x (1 + \tan^2 x) \sec^2 x dx =$$

$$\left. \begin{array}{l} u = \tan x \\ du = \sec^2 x \end{array} \right\} \rightarrow \int u^4 (1 + u^2) du = \int u^4 + u^6 du = \frac{u^5}{5} + \frac{u^7}{7} + C$$

$$= \frac{1}{5} \tan^5 x + \frac{1}{7} \tan^7 x + C$$

$$\int_0^{\pi/6} \tan^4 x \sec^4 x dx = \left(\frac{1}{5} \tan^5 x + \frac{1}{7} \tan^7 x \right) \Big|_0^{\pi/6} = \frac{1}{5} \left(\frac{1}{\sqrt{3}} \right)^5 + \frac{1}{7} \left(\frac{1}{\sqrt{3}} \right)^7 = \frac{26}{945\sqrt{3}}$$

(this level of simplification is OK!)

#10

$$a) \int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C$$

$$b) \int e^{-x} \sin x dx = -e^{-x} \sin x + \int e^{-x} \cos x dx$$

$$= -e^{-x} \sin x - e^{-x} \cos x + \int e^{-x} \sin x dx$$

$$\Rightarrow 2 \int e^{-x} \sin x dx = -e^{-x} (\sin x + \cos x) + C$$

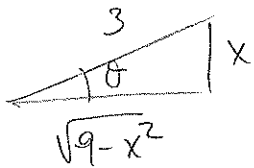
$$\Rightarrow \int e^{-x} \sin x dx = -\frac{e^{-x}}{2} (\sin x + \cos x) + C$$

$$\begin{aligned}
 c) \int x^2 \sin 4x \, dx &= x^2 \left(-\frac{\cos 4x}{4} \right) + \int \frac{2x \cdot \cos 4x}{4} \, dx \\
 &= -\frac{1}{4} x^2 \cos 4x + \frac{1}{2} \int x \cos 4x \, dx \\
 &= -\frac{1}{4} x^2 \cos 4x + \frac{1}{2} \left(x \frac{\sin 4x}{4} - \int \frac{\sin 4x}{4} \, dx \right) \\
 &= -\frac{x^2}{4} \cos 4x + \frac{x}{8} \sin 4x + \frac{1}{8} \frac{\cos 4x}{4} + C \\
 &= \boxed{\left(\frac{1}{32} - \frac{x^2}{4} \right) \cos 4x + \frac{x}{8} \sin 4x + C}
 \end{aligned}$$

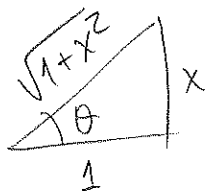
$$d) \int \ln(4x) \, dx = x \ln 4x - \int \frac{4x}{4x} \, dx = \boxed{x \ln 4x - x + C}$$

#11

$$\begin{aligned}
 a) \int \frac{x \, dx}{\sqrt{9-x^2}} &= \int \frac{3 \sin \theta \cdot 3 \cos \theta \, d\theta}{3 \cos \theta} = 3 \int \sin \theta \, d\theta = -3 \cos \theta + C \\
 \left. \begin{aligned} x &= 3 \sin \theta \\ dx &= 3 \cos \theta \, d\theta \end{aligned} \right\} &= -3 \frac{\sqrt{9-x^2}}{3} + C = \boxed{-\sqrt{9-x^2} + C}
 \end{aligned}$$

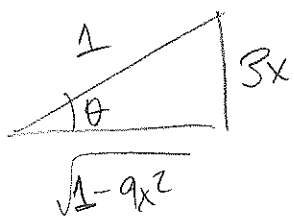


$$\begin{aligned}
 b) \int \frac{dx}{\sqrt{1+x^2}} &= \int \frac{\sec^2 \theta \, d\theta}{\sec \theta} = \int \sec \theta \, d\theta = \ln |\sec \theta + \tan \theta| + C \\
 \left. \begin{aligned} x &= \tan \theta \\ dx &= \sec^2 \theta \, d\theta \end{aligned} \right\} &= \ln |\sqrt{1+x^2} + x| + C \\
 &= \text{Note: } = \operatorname{arcsinh}(x) + C
 \end{aligned}$$



$$c) \int \sqrt{1-9x^2} dx = \int \sqrt{1-\sin^2\theta} \frac{\cos\theta d\theta}{3} = \frac{1}{3} \int \cos^2\theta d\theta$$

$$\left. \begin{array}{l} 3x = \sin\theta \\ 3dx = \cos\theta d\theta \end{array} \right\} \begin{array}{l} = \frac{1}{3} \int \frac{1+\cos 2\theta}{2} d\theta = \frac{1}{6} \left(\theta + \frac{\sin 2\theta}{2} \right) + C \\ = \frac{1}{6} \left(\arcsin 3x + 3x \cdot \sqrt{1-9x^2} \right) + C \end{array}$$



$$= \frac{1}{6} \arcsin 3x + \frac{x\sqrt{1-9x^2}}{2} + C$$

$$d) \int \frac{dx}{\sqrt{4x-x^2}} \xrightarrow{\text{complete the square}} \int \frac{dx}{\sqrt{4-(x-2)^2}} \xrightarrow{x-2=2\sin\theta} \int \frac{2\cos\theta d\theta}{2\cos\theta} = \int d\theta = \theta + C$$

$$\left. \begin{array}{l} \text{complete} \\ \text{the square} \end{array} \right\} \begin{array}{l} x-2=2\sin\theta \\ dx=2\cos\theta d\theta \end{array}$$

$$= \arcsin\left(\frac{x-2}{2}\right) + C$$

#12

$$a) \frac{1}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3} = \frac{A(x+3) + B(x-1)}{x^2+2x-3} \Rightarrow A(x+3) + B(x-1) = 1$$

$$A = 1/4, B = -1/4$$

$$\Rightarrow (A+B)x + 3A - B = 1$$

$$A = -B, 3A - B = 1$$

$$4A = 1$$

$$\int_2^3 \frac{dx}{x^2+2x-3} = \frac{1}{4} \int_2^3 \frac{dx}{x-1} - \frac{1}{4} \int_2^3 \frac{dx}{x+3} = \frac{1}{4} \left(\ln|x-1| - \ln|x+3| \right) \Big|_2^3 =$$

$$= \frac{1}{4} \ln\left(\frac{x-1}{x+3}\right) \Big|_2^3 = \frac{1}{4} \left(\ln \frac{2}{6} - \ln \frac{1}{5} \right) = \frac{1}{4} \ln \frac{5}{3}$$