Homework Set 9

DUE: APR 11 - 13, 2018 (AT THE BEGINNING OF RECITATION)

1. Find the Taylor Series of the following functions $f(x)$ centered at x_0:
   (a) $f(x) = x^4 - 2x^3 + x - 2$ at $x_0 = 0$
   (b) $f(x) = x^4 - 2x^3 + x - 2$ at $x_0 = 1$
   (c) $f(x) = \cos(3x^2)$ at $x_0 = 0$
   (d) $f(x) = \frac{1}{2}\ln(2x+1)$ at $x_0 = 0$

2. Use differentiation term-by-term to find the Taylor Series of $f'(x)$ centered at x_0 for each of the items in the previous exercise.

3. Compute the sum of the following series by recognizing it as the Taylor Series of an appropriate function:
   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}$
   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$
   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{2^n (2n+1)!}$
   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n} \pi^{2n}}{(2n)!}$

4. The goal of this exercise is to derive the so-called *Leibniz formula* for π, namely

$$\pi = 4\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \dots.$$

   (a) Write the Maclaurin Series[1] of the function $f(x) = \dfrac{1}{1+x^2}$
   (b) Use integration term-by-term to find the Maclaurin Series of $F(x) = \arctan x$
   (c) Evaluate $F(1)$ using the series obtained in (b) to prove the Leibniz formula for π

[1] Recall the Maclaurin Series of $f(x)$ is simply the Taylor Series of $f(x)$ centered at $x_0 = 0$.