MAT330/681 Lecture 1 1/31/2022

- Introduction, Course Syllabus, Q & A
- Motivating problems

True and False Positives

Version 1:
- The probability of having diabetes is 1%
- If someone has diabetes, there is a 90% prob. they test positive
- If someone does not have diabetes, the prob they nevertheless test positive is 9% "false positives"
- Someone tests positive. What is the prob. they have diabetes?

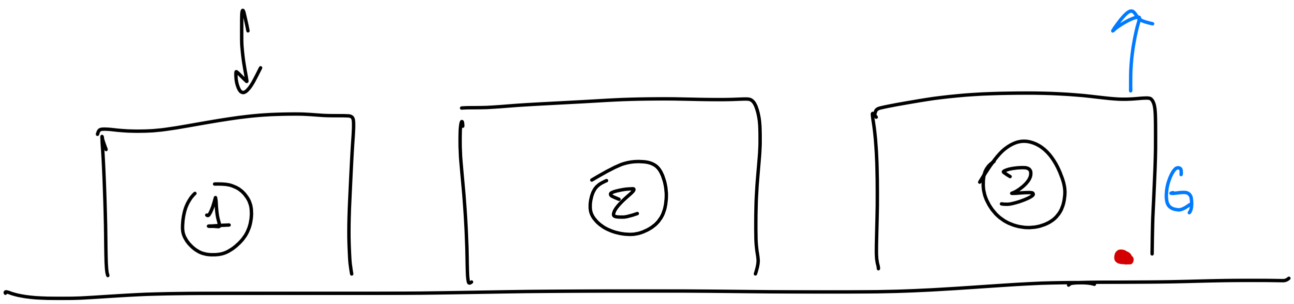
Most frequent answer among M.D.'s: 80% - 90%

Version 2:
- 10 in every 1000 people have diabetes 1%
- Of these 10, 9 will test positive 90%
- Of the 990 without diabetes, about 89 will nevertheless test positive.
- Someone tests positive. What is the prob. they have diabetes?

$$\frac{\overset{\text{true}}{9}}{\underset{\text{false positives}}{89}} \simeq 10.11\% < 50\%$$

$\boxed{\text{Bayes' Theorem}}$

Monty Hall Problem / "Let's make a deal"



C car

G goat

• = host reveals that door

your choice

"Stay"

1	2	3	result
C	G	G •	win
G	C	G •	lose
G	G •	C	lose

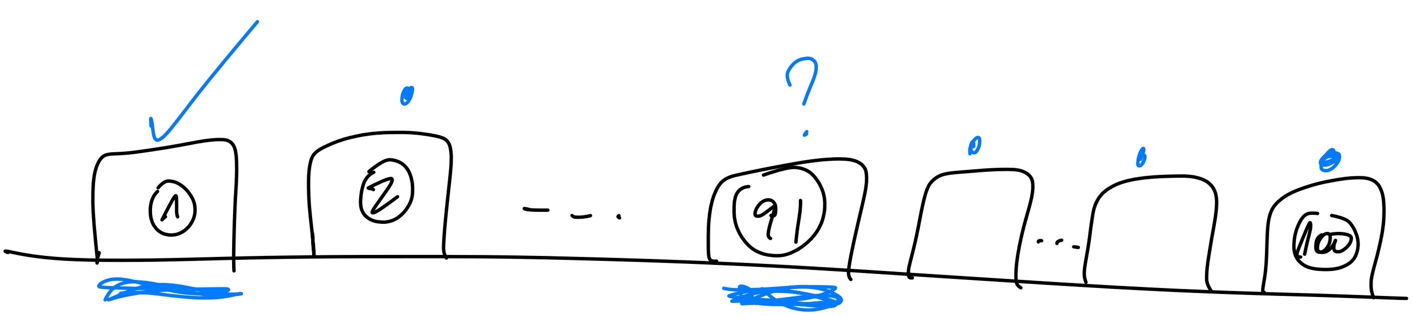
1st choice

"Switch"

1	2	3	result
C	G F.C.	G •	lose
G	C F.C.	G •	win
G	G •	C F.C.	win

F.C. = Final choice.

Win 1/3 of the time $\xrightarrow{2\times}$ Win 2/3 of the time



Basic Counting

Q: Lehman's cafeteria has the following lunch choices

3 options of sides	2 option of protein	5 options of drink

How many possible meals can be chosen?

$$\underline{3}\ \underline{2}\ \underline{5} = 30 \text{ meals.}$$

"Basic Principle of Counting"

Experiment 1: n_1 outcomes
Experiment 2: n_2 outcomes
⋮
Experiment k: n_k outcomes

Collection of outcomes for all experiments has a total of
$n_1 \cdot n_2 \cdot \cdots \cdot n_k$
possible outcomes.

License plates:

$$\underbrace{\underline{L}\ \underline{E}\ \underline{H}}_{\text{letters}}\ \underbrace{\underline{2}\ \underline{0}\ \underline{2}\ \underline{2}}_{\text{numbers}}$$

"Experiment" letter: 26 outcomes
"Experiment" digit: 10 outcomes

$$\text{total} = 26^3 \cdot 10^4 = 175{,}760{,}000$$

Permutations:

"In how many ways can you reshuffle n objects?"

WORD

ORWD

DROW

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

$$\underbrace{R}_{4} \cdot \underbrace{D}_{3} \cdot \underbrace{W}_{2} \cdot \underbrace{O}_{1} = 24$$

PEPPER 6 letters

P E _ _ _ _

2 E's

3 P's

$E_1 E_2$ P

P_1 P_2 P_3

$$\frac{6!}{2!\,3!} = ?$$

$$\frac{6!}{2!\,3!} = \frac{\overset{3}{\cancel{6}} \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{2!}\,\cancel{3!}} = 60$$

n objects

n_1 of them are alike

n_2 of them are alike

$\vdots$

n_k of them are alike

Total number of permutations:

$$\frac{n!}{n_1!\, n_2! \cdots n_k!}$$

Combinations: "In how many ways can you choose k objects from n possible objects, in no particular order?"

If the order matters

$$\underbrace{n}_{1^{st}} \quad \underbrace{(n-1)}_{2^{nd}} \quad \underbrace{(n-2)}_{3^{rd}} \quad \cdots \quad \underbrace{n-(k-1)}_{k^{th}\text{ choice}}$$

$$= n\cdot(n-1)\cdot\ldots\cdot(n-k+1) = \frac{n!}{(n-k)!}$$

If the order does not matter:

$$\frac{n\cdot(n-1)\cdot\ldots\cdot(n-k+1)}{k!} = \frac{n!}{(n-k)!\,k!}$$

\# of orders in which the choices take place

read this as "n choose k"

$$\binom{n}{k} := \frac{n!}{k!\,(n-k)!} \qquad \text{binomial coefficient}$$

$$\binom{4}{2} = \frac{4!}{2!\,2!} = \frac{24}{4} = \underline{\underline{6}}$$

Ex: Suppose you have 10 almonds and 5 cashew nuts in a bowl.

In how many ways can you pick them up 1 by 1 to eat?

15 nuts

C	A	A	C	A		C
1	2	3	4	...		15

$$\binom{15}{5} = \frac{15!}{10!\,5!} = \binom{15}{10}$$