MAT330/681, Spring 2022

Practice Problems for the Final Exam

1. In how many different ways can you paint a 3×3 checkerboard (i.e., a grid with 9 squares) using red, green, yellow, and blue ink, in such way that 2 squares are red, 2 squares are green, 3 squares are yellow, and the remaining squares are blue?

2. a) How many different words can be written using the letters in "SUMMER"?
   b) How many if all vowels must appear together (e.g., such as "SUEMMR")?
   c) What is the probability that a uniformly random word written using the letters in the word "SUMMER" is such that both vowels appear together?

3. Suppose you have a fair coin, i.e., a coin that flips to H (head) 50% of the time and to T (tail) 50% of the time.
   a) If you flip the coin 100 times, what is the probability you get 100 heads?
   b) After flipping the coin 100 times and getting 100 heads, you will now flip the coin once more. What is the probability that it flips to H?
   c) If you flip the coin 100 times, what is the probability you get 50 heads (in any order)?
   d) You are asked to play a game in which you choose a number $n \geq 2$ of times to flip this coin, and you win if and only if you flip to H exactly twice (and hence flip to T all $n-2$ other times). What choice of n maximizes the chance of winning?

4. Birdwatchers estimate that the probability of seeing a certain type of finch on any given day of bird watching is p. What is the probability that a birdwatcher sees this type of finch at least 7 times in the next 10 days? (Your answer should be in terms of p.)

5. Let X be a discrete random variable that assumes values 1, 4, and 5 with probabilities 1/4, 1/2, and 1/4, respectively. Compute the following:
   a) $P(X > 3) =$
   b) $P(X > 3 \,|\, X > 2) =$
   c) $P(X > 2 \,|\, X > 3) =$
   d) $E(X) =$
   e) $E(X^2) =$
   f) $\mathrm{Var}(X) =$
   g) $E(\sqrt{X}) =$

6. A survey of sport activities among Lehman College students reveals that:

   - 40% of students play basketball regularly;
   - 30% of students rides a bike regularly;
   - 25% of students swim regularly.

   Assume that engaging in the above sport activities are independent but not mutually exclusive events. What is the probability that a student does at least one of these activities, i.e., either plays basketball or rides a bike or swims regularly?

7. Suppose a farmer uses a plant fertilizer, and knows there were 2 bags of fertilizer in the barn, one is old and expired and the other one is new. Unfortunately the labels with the expiration dates got mixed up, so the farmer cannot tell which is which anymore, but the farmer knows that plants will grow with probability 2/3 if the non-expired fertilizer is used, and with probability 1/3 if the expired fertilizer is used. The farmer chooses one of them uniformly at random to use on 10 plants, and observes that exactly 4 of them grow. What is the probability that the non-expired fertilizer was used?

8. Suppose that the fine for speeding on a certain highway is \$200 for each occurrence, but 10% of drivers going over the speed limit are not caught by the highway patrol. What is the total cost in fines that a commuter taking this highway and going over the speed limit for 5 consecutive days should expect to incur?

9. The number of packages that arrive at a FedEx distribution center is modeled by a Poisson random variable, with an average of 65 packages arriving every 10 minutes. What is the probability that exactly 4 packages arrive in a given interval of 2 minutes?

10. The duration of each solar panel installed in a satellite orbiting the Earth until some failure renders it unusable is modeled by an exponential random variable whose mean is 200 months. If this particular model of satellite is powered by 3 independent solar panels and needs at least 1 of them to function in order to remain operational, what is the probability that the satellite will remain operational for at least 100 months?

11. Suppose that you choose (independently) two real numbers X and Y uniformly at random between 0 and 1. What is the probability that $X^2 < 5Y$?

12. Suppose that a random variable X has hazard rate function given by $3t^2$. Find the probability density function $f(x)$ of X.

13. Suppose that passwords for a certain website are strings consisting of letters, numbers, and special characters, and, on average, have 6 letters, 2 numbers, and 2 special

characters. Assume that the quantity of letters, numbers, and special characters are independent random variables.

a) Use Markov's inequality to give an upper bound for the probability that a given password consists of at least 8 letters, 3 numbers, and 3 special characters.

b) If the standard deviation for the number of letters in passwords on this website is 1, use Chebyshev's inequality to estimate from below the proportion of passwords that contain between 3 and 9 letters.

For the last 2 problems, use the table in the next page to approximate the values of $\Phi(a) = P(Z < a)$*, where* $Z \sim \text{Normal}(0, 1)$ *is a standard normal random variable.*

14. An ultrafast trade is a type of investment conducted automatically by a computer, that buys and then sells a certain stock within a single second. Suppose that a high frequency trading firm programs a computer to conduct a certain ultrafast trade 100 times. The expected return of each such trade is \$2, with a standard deviation of \$0.50. Use the Central Limit Theorem and the table below to approximate the probability that the total return from this operation is at least \$190.

15. Several medical trials are conducted to measure the half life (in hours) of a new drug after oral administration. Due to different metabolic properties of patients, the results of each measurement vary, but have common average $\mu = 5$, and common variance $\sigma^2 = 9$. Use the Central Limit Theorem to estimate how many patients should be used in the trial if the scientists need to know whether their measurement is accurate within 30 minutes with certainty of at least 92%.

Table 5.1 Area $\Phi(x)$ Under the Standard Normal Curve to the Left of X.

X	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998