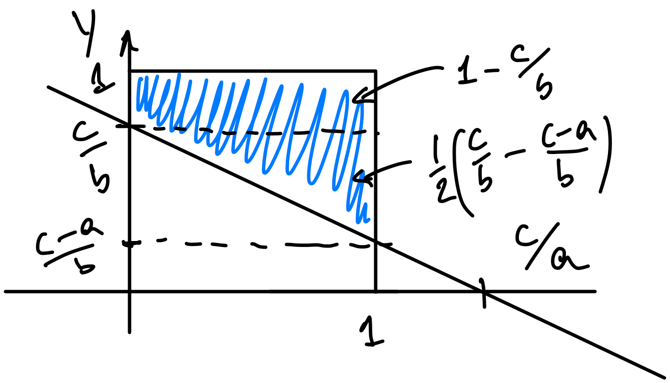
Solutions to HW 9

these numbers change.

#1 $\quad X, Y \sim \text{Uniform}(0,1), \qquad a>0,\ b>0,\ c>0$

$$P(aX+bY-c>0) = P\left(Y > \frac{c}{b} - \frac{a}{b}X\right)$$



$$= \left(1 - \frac{c}{b}\right) + \frac{1}{2}\left(\frac{c}{b} - \frac{c-a}{b}\right)$$

$$= \boxed{1 - \frac{c}{b} + \frac{a}{2b}}$$

#2 $\quad X \sim \text{Exponential}(\lambda)$

$$Y = aX^2 + b$$

these numbers change

i.e. $\quad Y = g(X) \quad$ where $\quad g(x) = ax^2 + b; \qquad a, b > 0$

$$P(X \geq x) = e^{-\lambda x}$$

$$F_Y(y) = P(Y \leq y) = P(aX^2 + b \leq y) = P\left(X^2 \leq \frac{y-b}{a}\right)$$

$$X \geq 0 \quad \overset{\curvearrowright}{=} P\left(X \leq \sqrt{\frac{y-b}{a}}\right) = 1 - P\left(X \geq \sqrt{\frac{y-b}{a}}\right)$$

$$= 1 - e^{-\lambda\sqrt{\frac{y-b}{a}}}$$

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy}\left(1 - e^{-\lambda\sqrt{\frac{y-b}{a}}}\right) = \frac{\lambda e^{-\lambda\sqrt{\frac{y-b}{a}}}}{2a\sqrt{\frac{y-b}{a}}}$$

$$f_Y(10) = \frac{\lambda e^{-\lambda\sqrt{\frac{10-b}{a}}}}{2\sqrt{a(10-b)}} \overset{\lambda=1/2}{=} \frac{e^{-\frac{1}{2}\sqrt{\frac{10-b}{a}}}}{4\sqrt{a(10-b)}}$$

#3 $L \sim \text{Lognormal}(0,1)$, i.e., $L = e^{Z}$, $Z \sim \text{Normal}(0,1)$.

$$E(aL+b) = ae^{1/2} + b = a\sqrt{e} + b \cong 1.65a + b.$$