Solutions to HW6

#1 $p = \frac{N}{100}$ ← this number changes each time.

X = # of members of household that develop the disease $\sim \text{Binomial}\left(5, \frac{N}{100}\right)$

$$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \binom{5}{0} p^0 (1-p)^5 + \binom{5}{1} p^1 (1-p)^4 + \binom{5}{2} p^2 (1-p)^3$$

$$= \left(1 - \frac{N}{100}\right)^5 + 5\,\frac{N}{100}\left(1-\frac{N}{100}\right)^4 + 10\,\frac{N^2}{100^2}\left(1-\frac{N}{100}\right)^3$$

$$= \left(1-\frac{N}{100}\right)^3\left(1 + \frac{3N}{100} + \frac{6N^2}{100^2}\right).$$

#2 Let X_{blue} and X_{red} be random variables for each fair die.

Recall from Lecture 10:

$$E(X_{blue}^n) = E(X_{red}^n) = \frac{1^n + 2^n + 3^n + 4^n + 5^n + 6^n}{6}$$

The expected income is the expected value of $aX_{blue}^n - bX_{red}^m$:

$$E\left(aX_{blue}^n - bX_{red}^m\right) = aE(X_{blue}^n) - bE(X_{red}^m)$$

$$= a\,\frac{1^n + 2^n + 3^n + 4^n + 5^n + 6^n}{6}$$

$$-b\,\frac{1^m + 2^m + 3^m + 4^m + 5^m + 6^m}{6}.$$

#3 $X \sim \text{Binomial}(n,p) \implies \sigma_X = \sqrt{Var(X)} = \sqrt{n\,p\,(1-p)}$.