Solutions to HW 4

#1 Consider the events:

I = interchanged card is selected

S = same value observed.

$$P(S) = P(S|I)P(I) + P(S|I^c)\cdot P(I^c)$$

$$= 1 \cdot \frac{1}{27} + \frac{3}{51}\,\frac{26}{27} \cong \underline{9.37\%}$$

only 3 other cards (of the 51 left) have same value as the interchanged one

$I \Rightarrow S$

second stack has 27 cards after receiving 1 from first stack

reloads w/ new numbers each time

#2

$P(\text{orange}) = A\ \%$

$P(\text{mango}) = B\ \%$

$P(\text{apple}) = C\ \%$

$P(\text{grape}) = \left(100 - (A+B+C)\right)\ \%$

Prob. of picking mango before orange is

$$\frac{P(\text{mango})}{P(\text{mango}) + P(\text{orange})} = \boxed{\frac{B}{A+B}}$$

From Lecture 7 (videos 3, 4)

reloads w/ new numbers each time

#3

200 slot machines

In 100 of them you win A% of the time "type A"

In the other 100 of them you win B% of the time. "type B"

You choose 1 machine at random and play 10 times, winning exactly 4 times.

H = You played the machine type A ← hypothesis

E = Win 4 out of 10 times. ← evidence.

From Bayes' formula; as discussed in Lecture 6:

$$P(H|E) = \frac{P(E|H)\overbrace{P(H)}^{=1/2}}{P(E|H)\underbrace{P(H)}_{=1/2} + P(E|H^c)\underbrace{P(H^c)}_{=1/2}}$$

$P(H) = P(H^c) = \frac{1}{2}$ b/c there are 100 type A and 100 type B machines

$$= \frac{P(E|H)}{P(E|H) + P(E|H^c)}$$

Prob. of 4 successes in 10 trials of Bernoulli process w/ $p = A/100$ (type A machine)

$$= \frac{\binom{10}{4}\left(\frac{A}{100}\right)^4\left(\frac{100-A}{100}\right)^6}{\binom{10}{4}\left(\frac{A}{100}\right)^4\left(\frac{100-A}{100}\right)^6 + \underbrace{\binom{10}{4}\left(\frac{B}{100}\right)^4\left(\frac{100-B}{100}\right)^6}}$$

Prob. of 4 successes in 10 trials of Bernoulli process w/ $p = B/100$ (type B machine)

$$= \boxed{\frac{A^4(100-A)^6}{A^4(100-A)^6 + B^4(100-B)^6}}$$