Solutions to HW 12

#1 $X =$ production

these numbers change.

$\mu = E(X)$

$\sigma^2 = Var(X)$

one-sided Chebyshev.

$$P(X \geq a) = P(X - \mu \geq a - \mu) \leq \frac{\sigma^2}{(a-\mu)^2 + \sigma^2}$$

#2 $S_n := X_1 + \cdots + X_n$, $\quad E(X_i) = \frac{7}{2}$, $\quad Var(X_i) = \frac{35}{12}$

each of the die

these numbers change

integer valued

$$P(A \leq S_n \leq B) = P(A - 0.5 \leq S_n \leq B + 0.5)$$

$\overline{X}_n = \frac{S_n}{n}$

$$= P\left(\frac{A-0.5}{n} \leq \overline{X}_n \leq \frac{B+0.5}{n}\right)$$

$$= P\left(\frac{\frac{A-0.5}{n} - \mu}{\sigma/\sqrt{n}} \leq \frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}} \leq \frac{\frac{B+0.5}{n} - \mu}{\sigma/\sqrt{n}}\right)$$

$a = \dfrac{\frac{B+0.5}{n} - \mu}{\sigma/\sqrt{n}} = -\dfrac{\frac{A-0.5}{n} - \mu}{\sigma/\sqrt{n}}$

b/c $\frac{A+B}{2} = n\mu = \frac{7}{2}n$

$$\overset{CLT}{\cong} P\left(-a \leq \frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}} \leq a\right)$$

$$= \Phi(a) - \Phi(-a) = 2\Phi(a) - 1$$

Your answer is the number

$$a = \frac{\frac{7}{2} - \frac{A-0.5}{n}}{\sqrt{\frac{35}{12n}}}$$

#3 $X_1, \longrightarrow X_n$ iid., $E(X_i) = \mu$, $Var(X_i) = \sigma^2$.

these numbers change...

$$\overline{X}_n = \frac{X_1 + \cdots + X_n}{n}$$

$$E(\overline{X}_n) = E\left(\frac{X_1 + \cdots + X_n}{n}\right) = \frac{E(X_1) + \cdots + E(X_n)}{n} = \frac{\cancel{n} \cdot \mu}{\cancel{n}} = \boxed{\mu.}$$

$$Var(\overline{X}_n) = Var\left(\frac{X_1 + \cdots + X_n}{n}\right) \overset{iid}{=} \frac{Var(X_1) + \cdots + Var(X_n)}{n^2}$$

$$\overset{iid}{=} \frac{\sigma^2 + \cdots + \sigma^2}{n^2} = \frac{n\sigma^2}{n^2} = \boxed{\frac{\sigma^2}{n}}$$

<u>Answer</u>: $E(\overline{X}_n) + Var(\overline{X}_n) = \mu + \frac{\sigma^2}{n}$.