Solutions to HW10

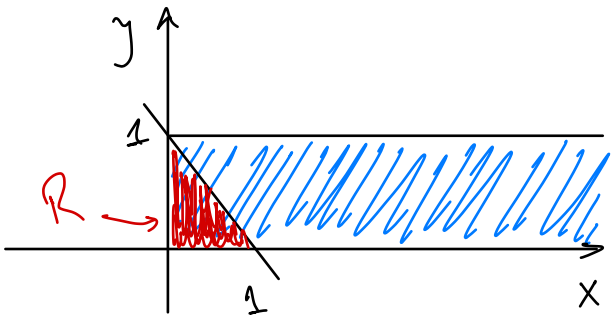
#1 $X \sim \text{Exponential}(\lambda)$ ← this changes

$Y \sim \text{Uniform}([0,1])$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} 1 & \text{if } \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

$$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y) = \begin{cases} \lambda e^{-\lambda x} & \text{if } (x,y) \in [0,+\infty) \times [0,1] \\ 0 & \text{otherwise} \end{cases}$$



$y = 1 - x \iff x = 1 - y$

$R = \{(x,y) : 0 \leq y \leq 1, \quad 0 \leq x \leq 1-y\}$.

$$P(X+Y \leq 1) = P(Y \leq 1 - X)$$

$$= \iint_R f_{X,Y}(x,y)\,dx\,dy$$

$$= \int_0^1 \int_0^{1-y} \lambda e^{-\lambda x}\,dx\,dy$$

$$= \int_0^1 \left(-e^{-\lambda x}\right)\Big|_0^{1-y} dy$$

$$= \int_0^1 1 - e^{-\lambda(1-y)}\,dy$$

$$= 1 - e^{-\lambda} \int_0^1 e^{\lambda y}\,dy$$

$$= 1 - e^{-\lambda}\left(\frac{e^{\lambda y}}{\lambda}\right)\Big|_0^1$$

$$\boxed{= 1 - \frac{1}{\lambda} + \frac{e^{-\lambda}}{\lambda}}$$

#2 X = output from solar panel that costs \$4,000

Y = " " " " \$3,000

Z = " " " " \$2,500

$$40 \le E(aX+bY+cZ) = aE(X)+bE(Y)+cE(Z)$$

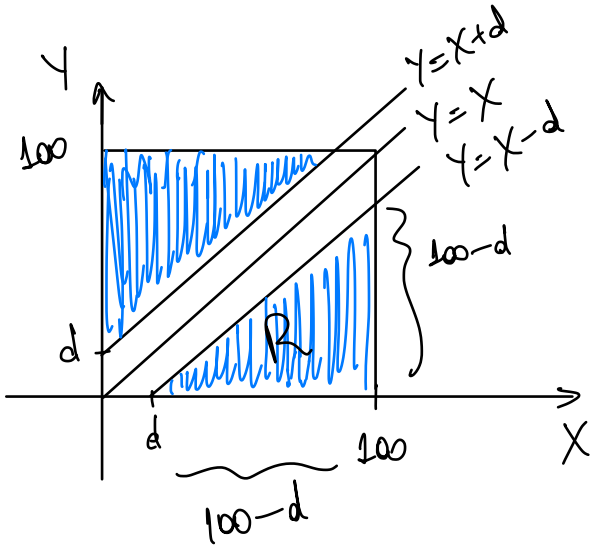
these change. ↑ unknown

$$E(Z) \ge \frac{40 - aE(X) - bE(Y)}{c}$$

#3 X = location of migrants

Y = location of patrol

$|Y-X|$ = distance between them



this changes

$$P(|Y-X| \ge d) = P(-d \le Y-X \le d)$$

$$= P(X-d \le Y \le X+d)$$

$$= \frac{2 \cdot \text{Area}(R)}{100^2}$$

$$= \boxed{\frac{(100-d)^2}{100^2}}$$