

Homework Set 6

DUE: NOV 22, 2021 (VIA BLACKBOARD, BY 11.59PM)

To be handed in:*Please remember that all problems will be graded!*

1. Consider the sequence of functions $f_n(x) = \frac{n + \cos x}{3n + \sin^2 x}$ for all $x \in \mathbb{R}$.
 - (a) Find an explicit function $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $(f_n)_{n \in \mathbb{N}}$ converges uniformly on \mathbb{R} to f . You must justify why the convergence is uniform by verifying the definition of uniform convergence.
 - (b) Use the function $f(x)$ to compute $\lim_{n \rightarrow +\infty} \int_1^5 f_n(x) dx$.

2. (a) Use differentiation term-by-term and an example from class (Lectures 15/18) to prove that $\sum_{n=1}^{+\infty} nx^n = \frac{x}{(1-x)^2}$ for all $|x| < 1$.
 - (b) Compute $\sum_{n=1}^{+\infty} \frac{n}{2^n}$.