MAT 330 Solution to HW8

1. $f(x) = \begin{cases} C \sin^2 x, & 0 \le x \le \pi \\ 0 & \text{otherwise} \end{cases}$ is a p.d.f.

if and only if $\int_{-\infty}^{+\infty} f(x)\,dx = 1$.

$$1 = \int_{-\infty}^{+\infty} f(x)\,dx = \int_0^{\pi} C \sin^2 x\,dx = C \int_0^{\pi} \frac{1-\cos 2x}{2}\,dx = C \cdot \frac{\pi}{2}$$

Therefore, $\boxed{C = \frac{2}{\pi}}$.

2. $$E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_0^{\pi} x \frac{2}{\pi} \sin^2 x\,dx =$$

$$= \frac{2}{\pi} \int_0^{\pi} x \frac{1-\cos 2x}{2}\,dx = \frac{1}{\pi} \int_0^{\pi} x - x\cos 2x\,dx$$

$$= \frac{1}{\pi} \cdot \left(\frac{x^2}{2}\Big|_0^{\pi} - \underbrace{x\frac{\sin 2x}{2}\Big|_0^{\pi}}_{=0.} + \underbrace{\int_0^{\pi} \frac{\sin 2x}{2}\,dx}_{=0.} \right)$$

$$= \frac{1}{\pi} \frac{\pi^2}{2} = \boxed{\frac{\pi}{2}}$$

3. $$E(\cos(X)) = \int_0^{\pi} \cos x \cdot \frac{2}{\pi} \sin^2 x\,dx = \frac{2}{\pi} \left(\frac{\sin^3 x}{3} \right)\Big|_0^{\pi}$$

$$= \frac{2}{\pi} \cdot 0 = \boxed{0.}$$