MAT330/681 Solutions to HW6

Define discrete random variables A and B, which are the earnings of playing game A and B respectively.

a) $E(A) = 1 \cdot P(A=1) + (-0.5)\, P(A=-0.5)$

$= 1 \cdot \frac{1}{2} + (-0.5)\frac{1}{2} = \frac{1-0.5}{2} = \frac{0.5}{2} = \boxed{0.25}$

b) $E(B) = 5 \cdot P(B=5) + (-6)\, P(B=-6)$

$= 5 \cdot \frac{1}{2} + (-6) \cdot \frac{1}{2} = \frac{5-6}{2} = \boxed{-0.5}$

c) $E(A+B) \overset{\text{linearity}}{=} E(A) + E(B) = 0.25 - 0.5 = \boxed{-0.25}$

d) $Var(A) = \underbrace{E(A^2)}_{E(A^2) = 1^2 \cdot \frac{1}{2} + (-0.5)^2 \frac{1}{2} = \frac{1+0.25}{2} = 0.625} - E(A)^2 = 0.625 - (0.25)^2 = \boxed{0.5625}$

e) $Var(B) = \underbrace{E(B^2)}_{E(B^2) = 5^2 \frac{1}{2} + (-6)^2 \cdot \frac{1}{2} = \frac{25+36}{2} = \frac{61}{2} = 30.5} - E(B)^2 = 30.5 - 0.25 = \boxed{30.25}$

f) $Cov(A,B) = 0$ because A and B are <u>independent</u>.

g) $Var(A+B) = Var(A) + Var(B) + 2\,Cov(A,B)$

$= 0.5625 + 30.25 + 0 = \boxed{30.8125}$

h) $\boxed{\text{Game B is riskier}}$ because $Var(B) > Var(A)$.

i) $\boxed{\text{Game A is more profitable}}$ because $E(A) > E(B)$

(Actually Game A is the only profitable game, since $E(A) > 0$, while Game B has a negative expected value $E(B) < 0$!)

j) $\boxed{\text{Most profitable strategy is to only play A!}}$

SIDE COMMENT ABOUT j) FOR THOSE INTERESTED IN FINANCE:

In real life, you could also use B to create a profitable strategy by exploiting your knowledge of $E(B)$ and $Var(B)$, assuming your competitors have less knowledge of it; by placing convenient "bets" on outcomes of Game B without engaging in actually playing Game B. In finance, this is called a "derivative", more precisely, an option on the underlying asset B. To profit from B being very volatile (meaning $Var(B)$ is large) you can use option strategies called "straddles" or "strangles" which consist of a simultaneous call and put option on B. For more, look up these terms on Google or send me an e-mail.