

## Homework Set 4

DUE: OCT 21, 2020 (VIA BLACKBOARD, BY 11.59PM)

**To be handed in:***Please remember that all problems will be graded!*

1. Decide if each of the statements below is **true** or **false**. If it is true, give a complete **proof**; if it is false, give an explicit **counter-example**.

(a) If  $\sum_{n=1}^{\infty} a_n$  converges and  $a_n \geq 0$  for all  $n \in \mathbb{N}$ , then  $\sum_{n=1}^{\infty} \sqrt{a_n}$  converges.

(b) If  $\sum_{n=1}^{\infty} a_n$  converges and  $a_n \geq 0$  for all  $n \in \mathbb{N}$ , then  $\sum_{n=1}^{\infty} a_n^2$  converges.

(c) If  $\sum_{n=1}^{\infty} a_n$  converges and  $a_n \geq 0$  for all  $n \in \mathbb{N}$ , then  $\sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n}$  converges.

(d) If  $\sum_{n=1}^{\infty} a_n$  converges and  $a_n \geq 0$  for all  $n \in \mathbb{N}$ , then the power series  $\sum_{n=1}^{\infty} a_n x^n$  converges absolutely for all  $x \in [-1, 1]$ .

2. Use either the Root or Ratio test to find the radius of convergence of the following power series:

(a)  $\sum_{n=1}^{\infty} \frac{5^n}{n!} z^n$

(b)  $\sum_{n=1}^{\infty} \frac{7}{4^n} z^n$

(c)  $\sum_{n=1}^{\infty} \frac{2^n}{\sqrt{n}} z^n$