Name: ANSWERS Lehman ID: _____

By writing my name above, I acknowledge complying with the CUNY Academic Integrity Policy while completing this examination.

MAT 226 (Spring 2020)
Quiz 2

1. **(6 pts)** Compute the length $\|\gamma'(t)\|$ of the tangent vector to the curve

$$\gamma(t) = (\cos t + t\sin t)\,\mathbf{i} + \frac{t^2}{2}\mathbf{j} + (t\cos t - \sin t)\,\mathbf{k}$$

$$\gamma'(t) = (-\sin t + \sin t + t\cos t)\,\hat{i} + t\,\hat{j} + (\cos t - t\sin t - \cos t)\,\hat{k}$$

$$= t\cos t\,\hat{i} + t\,\hat{j} - t\sin t\,\hat{k}$$

$$\|\gamma'(t)\|^2 = t^2\cos^2 t + t^2 + t^2\sin^2 t = t^2\underbrace{(\cos^2 t + \sin^2 t)}_{1} + t^2$$

$$= 2t^2$$

$$\Rightarrow \boxed{\|\gamma'(t)\| = \sqrt{2}\cdot t} \quad \text{if} \quad t \geq 0$$

2. **(4 pts)** Use the above computation to find the arclength of $\gamma(t)$ from $t = 0$ to $t = 1$.

$$\ell_0^1(\gamma) = \int_0^1 \|\gamma'(t)\|\,dt = \int_0^1 \sqrt{2}\cdot t\,dt = \sqrt{2}\cdot\frac{t^2}{2}\Big|_0^1 = \boxed{\frac{\sqrt{2}}{2}}$$