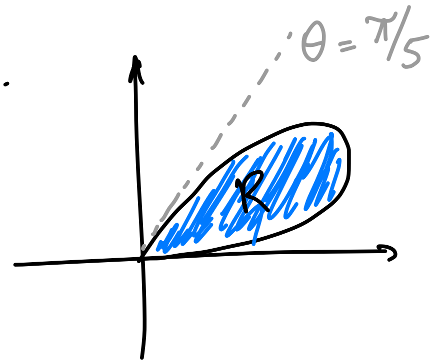
MAT226

Solution to HW 9

1.



$$r(\theta) = \sin(5\theta)$$

$$r(\theta) = 0 \iff 5\theta \in \{0, \pi, 2\pi, 3\pi, \dots\}$$

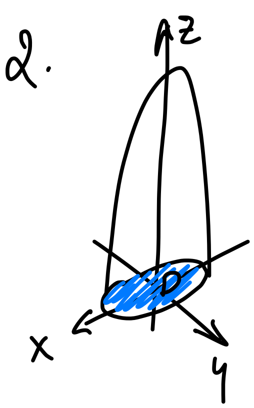
$$\iff \theta \in \left\{0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \dots\right\}$$

Petal: $0 \le \theta \le \frac{\pi}{5}$, $\quad 0 \le r \le \sin(5\theta)$

$$\text{Area} = \iint_R 1 \, dA = \int_0^{\pi/5} \int_0^{\sin 5\theta} r \, dr \, d\theta = \int_0^{\pi/5} \frac{r^2}{2}\Big|_0^{\sin 5\theta} d\theta$$

$$= \frac{1}{2} \int_0^{\pi/5} \sin^2 5\theta \, d\theta = \frac{1}{2} \int_0^{\pi/5} \frac{1 - \cos 10\theta}{2} \, d\theta = \frac{1}{4} \cdot \frac{\pi}{5} = \boxed{\frac{\pi}{20}}$$

2.



$$z = 8 - 2x^2 - 2y^2$$

$$D: \; 2(x^2 + y^2) = 8$$

$$x^2 + y^2 = 2^2$$

$$V = \int_0^{2\pi} \int_0^{2} (8 - 2r^2) \, r \, dr \, d\theta$$

$$= 2\pi \int_0^{2} 8r - 2r^3 \, dr$$

$$= 2\pi \left(4r^2 - \frac{r^4}{2}\right)\Big|_0^{2}$$

$$= 2\pi(16 - 8) = \boxed{16\pi}$$