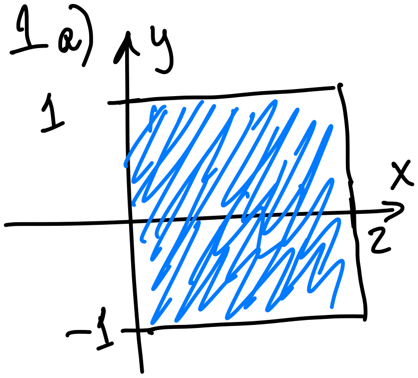
MAT226

Solution to HW8

1a)



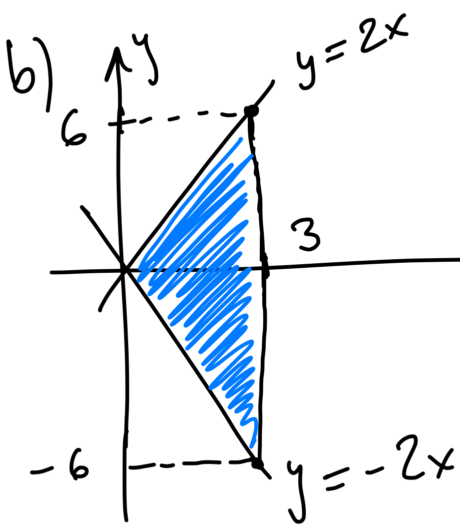
$$\int_{-1}^{1}\int_{0}^{2} x^3y + xy^2 - 1 \, dx\,dy =$$

$$= \int_{-1}^{1} \left(\frac{x^4}{4} y + \frac{x^2}{2} y^2 - x \right)\Bigg|_0^2 dy$$

$$= \int_{-1}^{1} 4y + 2y^2 - 2 \; dy = \left(2y^2 + \frac{2}{3}y^3 - 2y \right)\Bigg|_{-1}^{1}$$

$$= \cancel{2} + \frac{2}{3} - \cancel{2} - \left(2 - \frac{2}{3} + 2 \right) = \frac{4}{3} - 4 = \boxed{-\frac{8}{3}}$$

b)



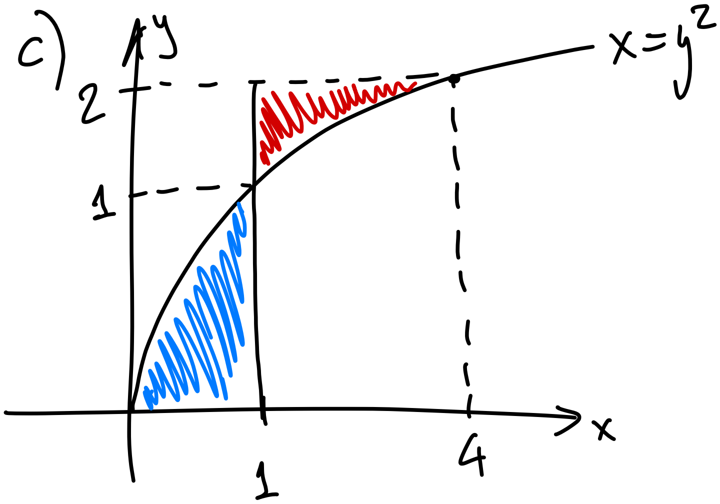
$$\int_0^3 \int_{-2x}^{2x} 5y - 3 \; dy\,dx =$$

$$= \int_0^3 \left(\frac{5y^2}{2} - 3y \right)\Bigg|_{-2x}^{2x} dx$$

$$= \int_0^3 \frac{5}{2} \cdot 4x^2 - 6x - \left(\frac{5}{2} \cdot 4x^2 + 6x \right) dx$$

$$= \int_0^3 -12x \, dx = -12 \frac{x^2}{2}\Bigg|_0^3 = \boxed{-54}$$

c)



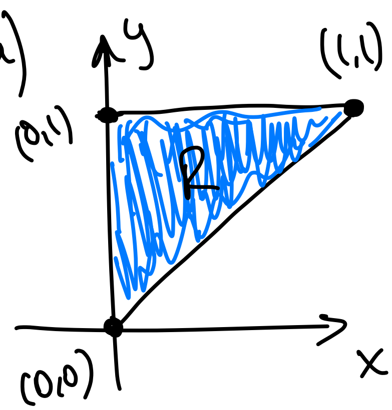
$$\int_0^2\int_{y^2}^1 ye^x\,dx\,dy =$$

$$=\int_0^2 \left(ye^x\Big|_{y^2}^1\right)dy$$

$$=\int_0^2 ye - ye^{y^2}\,dy = e\,\frac{y^2}{2}\Big|_0^2 - \frac{e^{y^2}}{2}\Big|_0^2$$

$$\boxed{= 2e - \frac{e^4}{2} + \frac{1}{2}}$$

2 a)



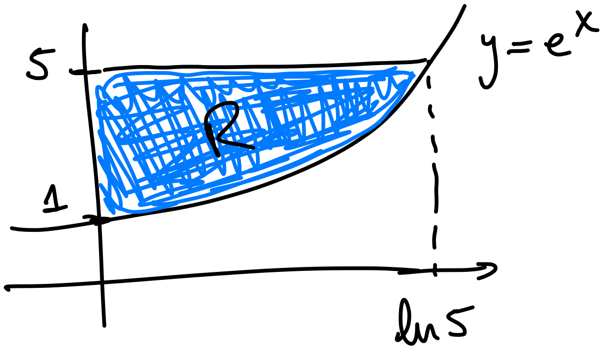
$R: \; 0 \le y \le 1, \quad 0 \le x \le y$

$$\iint_R \sqrt{1-y^2}\,dA = \int_0^1\int_0^y \sqrt{1-y^2}\,dx\,dy$$

$$=\int_0^1 \left(x\sqrt{1-y^2}\right)\Big|_0^y dy = \int_0^1 y\sqrt{1-y^2}\,dy$$

$$= -\frac{(1-y^2)^{3/2}}{3}\Big|_0^1 = \boxed{\frac{1}{3}}$$

⚠ Note: $dA = dy\,dx$ would lead to a much harder integral (w/ trig substitution...)

b) 

R: $1 \le y \le 5, \quad 0 \le x \le \ln y$

$$\iint_R \frac{1}{\ln y}\,dA = \int_1^5 \int_0^{\ln y} \frac{1}{\ln y}\,dx\,dy$$

$$= \int_1^5 \frac{x}{\ln y}\Big|_0^{\ln y} dy = \int_1^5 \left(\frac{\ln y}{\ln y} - \frac{0}{\ln y}\right) dy$$

$$= \int_1^5 1\,dy = 5 - 1 = \boxed{4}$$

! **Note:** The order of integration $dA = dy\,dx$ would not be feasible, since $\int \frac{1}{\ln y}\,dy$ cannot be computed in terms of elementary functions.