MAT 226 Solution to HW7

Target function: $f(x,y) = 3x - 4y$

Constraint: $g(x,y) = x^2 + y^2 = 4$

1. $\nabla f(x,y) = (3, -4)$

   $\nabla g(x,y) = (2x, 2y)$

2. $\nabla f(x,y) = \lambda \nabla g(x,y) \iff \begin{cases} 3 = \lambda \cdot 2x \\ -4 = \lambda \cdot 2y \end{cases}$

$$\Rightarrow \lambda = \frac{3}{2x} = -\frac{\overset{2}{\cancel{4}}}{\cancel{2}y} \Rightarrow 3y = -4x$$

$$\Rightarrow y = -\frac{4}{3}x$$

From constraint: $x^2 + \left(-\frac{4}{3}x\right)^2 = 4 \Rightarrow x^2 + \frac{16x^2}{9} = 4$

$$\Rightarrow \frac{25x^2}{9} = 4 \Rightarrow x^2 = \frac{36}{25} \Rightarrow x = \pm\frac{6}{5}$$

Since $y = -\frac{4}{3}x$, we get the candidate points:

$\left(\frac{6}{5}, -\frac{8}{5}\right)$ and $\left(-\frac{6}{5}, \frac{8}{5}\right)$

Maximum Value

3. $f\left(\frac{6}{5}, -\frac{8}{5}\right) = 3 \cdot \frac{6}{5} + 4\,\frac{8}{5} = \frac{18+32}{5} = \frac{50}{5} = \boxed{10}$

   $f\left(-\frac{6}{5}, \frac{8}{5}\right) = -3 \cdot \frac{6}{5} - 4 \cdot \frac{8}{5} = \boxed{-10}$

Minimum Value