MAT 226 Solution to HW6

Let $F(x,y,z) = \left(5 - \sqrt{x^2+y^2}\right)^2 + z^2$ and note that the given surface is the 1-levelset of $F(x,y,z)$. In other words, a point (x,y,z) belongs to the given surface if and only if $F(x,y,z)=1$. Thus, $\nabla F(x,y,z)$ is normal to the surface at that point.

$$\nabla F(x,y,z) = \left(2\left(5-\sqrt{x^2+y^2}\right)\left(-\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2}}\right), \right.$$

$$\left. 2\left(5-\sqrt{x^2+y^2}\right)\left(-\frac{\cancel{2}y}{\cancel{2}\sqrt{x^2+y^2}}\right), 2z \right)$$

At the point $(x_0,y_0,z_0) = \left(\frac{10+\sqrt{3}}{2}, 0, \frac{1}{2}\right)$, we have:

$$\nabla F\left(\frac{10+\sqrt{3}}{2}, 0, \frac{1}{2}\right) = \left(-2\left(5-\frac{10+\sqrt{3}}{2}\right)\frac{\cancel{\frac{10+\sqrt{3}}{2}}}{\cancel{\frac{10+\sqrt{3}}{2}}}, 0, 1\right)$$

$$= \left(-10+10+\sqrt{3}, 0, 1\right)$$

$$= \left(\sqrt{3}, 0, 1\right).$$

Thus, the equation of the tangent plane is:

$$\left\langle \left(x - \frac{10+\sqrt{3}}{2}, y, z - \frac{1}{2}\right), \left(\sqrt{3}, 0, 1\right) \right\rangle = 0$$

$$x\sqrt{3} - \frac{10\sqrt{3}+3}{2} + z - \frac{1}{2} = 0$$

$$x\sqrt{3} + z - \frac{10\sqrt{3}+4}{2} = 0$$

$$\boxed{x\sqrt{3} + z - 5\sqrt{3} - 2 = 0}$$