MAT 226 Solutions to HW 4

1. $$\int_0^{\pi} \frac{\cos t}{\sin t + 1}\,\hat{\imath} - t\,\hat{\jmath} + t e^t\,\hat{k}\; dt =$$

$$= \left(\ln|\sin t + 1|\right)\Big|_0^{\pi}\hat{\imath} - \frac{t^2}{2}\Big|_0^{\pi}\hat{\jmath} + \left(te^t - e^t\right)\Big|_0^{\pi}\hat{k}$$

$$= (\ln 1 - \ln 1)\,\hat{\imath} - \frac{\pi^2}{2}\,\hat{\jmath} + \left(\pi e^{\pi} - e^{\pi} + e^{0}\right)\hat{k}$$

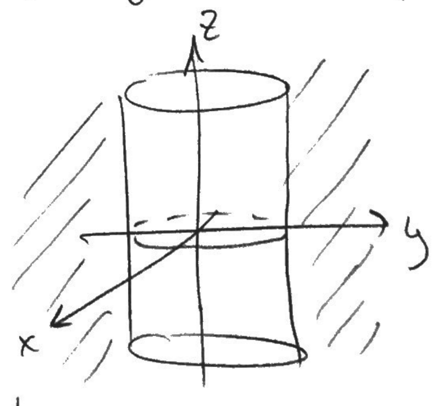
$$= \boxed{0\,\hat{\imath} - \frac{\pi^2}{2}\,\hat{\jmath} + \left((\pi - 1)e^{\pi} + 1\right)\hat{k}}$$

OR: $\left(0, -\frac{\pi^2}{2}, (\pi-1)e^{\pi} + 1\right)$.

2. $$\boxed{\text{Domain}(f) = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 - 1 > 0\}}$$

$$= \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 > 1\}$$

is the exterior of the (infinite) cylinder of radius 1 along the z-axis;



$$\boxed{\text{Image}(f) = (0, +\infty)}$$

since $w = \sqrt{\dfrac{e^z}{x^2+y^2-1}}$ has a solution for any $w > 0$.

E.g., take $x = \sqrt{2}$, $y = 0$, and $z = 2\ln w$, then

$$f(\sqrt{2}, 0, 2\ln w) = \sqrt{\frac{e^{2\ln w}}{2-1}} = \sqrt{w^2} = w$$