MAT 226

Solutions to HW3

1. $(x^2+y^2)^2 = 2(x^2-y^2)$

Substitute in: $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ (polar coordinates)

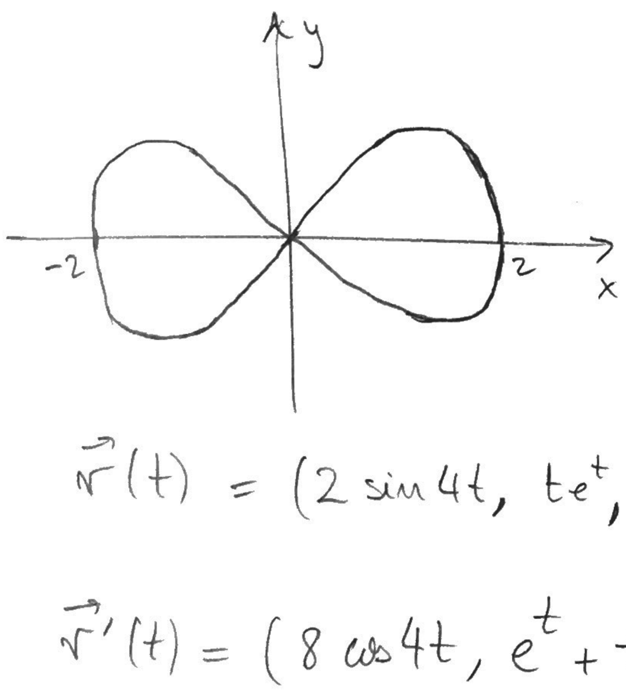
$$\left(r^2\cos^2\theta + r^2\sin^2\theta\right)^2 = 2\left(r^2\cos^2\theta - r^2\sin^2\theta\right)$$

$$\left(r^2\underbrace{\left(\cos^2\theta + \sin^2\theta\right)}_{1}\right)^2 = 2r^2\underbrace{\left(\cos^2\theta - \sin^2\theta\right)}_{\cos 2\theta}$$

$$r^4 = 2r^2\cos 2\theta$$

$$\boxed{r^2 = 2\cos 2\theta} \quad \text{or} \quad r(\theta) = \pm\sqrt{2|\cos 2\theta|}$$

Extra points: This curve is a lemniscate:



2. $\vec{r}(t) = (2\sin 4t,\ te^t,\ 1+\sqrt{t})$

a) $\vec{r}\,'(t) = \left(8\cos 4t,\ e^t + te^t,\ \frac{1}{2\sqrt{t}}\right)$

b) $\|\vec{r}\,'(t)\|^2 = 64\cos^2 4t + (1+t)^2 e^{2t} + \frac{1}{4t}$

c) $\langle \vec{r}\,'(t), \vec{r}(t)\rangle = 16\sin 4t\cos 4t + t(1+t)e^{2t} + \frac{1+\sqrt{t}}{2\sqrt{t}}$.