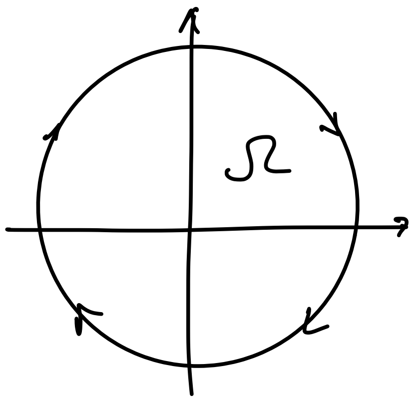
MAT226 Solution to HW11

1 a) $\vec{F}(x,y) = (\underbrace{x^2y}_{=M}, \underbrace{xy^2}_{=N})$ $\quad \dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y} = y^2 - x^2$ (not conservative)



$\gamma(t) = (\cos t, \sin t)$
$t \in [0, 2\pi]$

By Green's Thm:

$$\int_\gamma M\,dx + N\,dy = \iint_\Omega \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dA$$

$$= \int_0^{2\pi}\int_0^1 (r^2\sin^2\theta - r^2\cos^2\theta)\, r\,dr\,d\theta$$

$$= \underbrace{\left(\int_0^{2\pi}\left(-\frac{\cos 2\theta}{2}\right)d\theta\right)}_{=0}\underbrace{\left(\int_0^1 r^3\,dr\right)}_{=\frac{1}{4}} = \boxed{0.}$$

b) $\vec{F}(x,y,z) = (ze^y, 2x\sin z, x+z+1)$ not conservative (from HW 10)

$\gamma(t) = (t^2+1, t, 0)$, $t \in [0,1]$

$\gamma'(t) = (2t, 1, 0)$

$$\int_\gamma \vec{F}\,d\gamma = \int_0^1 \langle (0, 2(t^2+1)\underbrace{\sin 0}_{=0}, t^2+1+1), (2t, 1, 0)\rangle\, dt = \int_0^1 0\,dt = \boxed{0}$$

c) $\vec{F}(x,y) = (e^x\cos y, -e^x\sin y)$

From HW10: $\vec{F}$ is conservative and has potential

$f(x,y) = e^x\cos y + c$ for any $c \in \mathbb{R}$

$$\gamma(t) = \left(\left(1 + (-1)^t\right)\cos\left(t^6 - 4t^2 + \ln t\right), \pi t\right), \quad t \in [3,5]$$

$$\int_\gamma \vec{F}\, d\gamma \overset{\text{F.T.C}}{=} f(\gamma(5)) - f(\gamma(3)) = f(0, 5\pi) - f(0, 3\pi)$$

$$= e^0 \cos 5\pi - e^0 \cos 3\pi = -1 - (-1) = \boxed{0}$$