MAT226 Solution to HW 10

1. a) $\vec{F}(x,y) = (\underbrace{x^2 y}_{M}, \underbrace{xy^2}_{N})$, $\Omega = \mathbb{R}^2$

$$\frac{\partial M}{\partial y} = x^2 \neq \frac{\partial N}{\partial x} = y^2$$ so $\vec{F}$ is <u>not</u> conservative!

b) $\vec{F}(x,y,z) = (ze^y, 2x\sin z, x+z+1)$, $\Omega = \mathbb{R}^3$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ ze^y & 2x\sin z & x+z+1 \end{vmatrix}$$

$$= (-2x\cos z, -1+e^y, 2\sin z - ze^y) \neq (0,0,0)$$

so $\vec{F}$ is <u>not</u> conservative!

c) $\vec{F}(x,y) = (\underbrace{e^x \cos y}_{=M}, \underbrace{-e^x \sin y}_{=N})$, $\Omega = \mathbb{R}^2$

$$\left.\begin{array}{l} \dfrac{\partial M}{\partial y} = -e^x \sin y = \dfrac{\partial N}{\partial x} = -e^x \sin y \\ \Omega = \mathbb{R}^2 \text{ is simply-connected} \end{array}\right\} \Rightarrow \vec{F} \text{ is } \underline{\text{conservative}}!$$

To find a potential $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$ for $\vec{F}$:

$$f(x,y) = \int M \, dx + g(y) = \int e^x \cos y \, dx + g(y)$$

$$= e^x \cos y + g(y)$$

$$\frac{\partial f}{\partial y} = -e^x \sin y + g'(y) = N \Longrightarrow g'(y) = 0$$

$$\Longrightarrow g(y) = c.$$

So $f(x,y) = e^x \cos y$ is a potential for $\vec{F}$.