Name: ANSWERS Lehman ID: ______________

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MAT 226 (Fall 2019)
Quiz 6

Consider the vector field $\vec{F}(x,y,z) = (x^2, z+x, 2y^2)$ on the cube $\Omega = [0,1]\times[0,1]\times[0,1]$.

a) **(2pts)** Compute the **divergence** of $\vec{F}$.

$$\operatorname{div}\vec{F} = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(z+x) + \frac{\partial}{\partial z}(2y^2) = \boxed{2x}$$

b) **(2pts)** Compute the triple integral of $\operatorname{div}\vec{F}$ on the cube Ω.

$$\iiint_\Omega \operatorname{div}\vec{F}\,\mathrm{d}V = \int_0^1\int_0^1\int_0^1 2x\,dx\,dy\,dz = x^2\Big|_0^1 = \boxed{1}$$

c) **(3pts)** Compute the **curl** of $\vec{F}$.

$$\operatorname{curl}\vec{F} = \nabla\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 & z+x & 2y^2 \end{vmatrix} = \boxed{(4y-1,\,0,\,1)}$$

d) **(3pts)** Compute the line integral of $\operatorname{curl}\vec{F} = \nabla\times\vec{F}$ along the line segment γ that joins $P=(2,3,4)$ to $Q=(-1,0,1)$. [Note: begin by finding a **parametrization** of γ.]

$$\int_\gamma \operatorname{curl}\vec{F}\,\mathrm{d}\gamma = \int_\gamma \nabla\times\vec{F}\,\mathrm{d}\gamma =$$

$$\gamma(t) = t(-1,0,1) + (1-t)(2,3,4) = (-t,0,t) + (2-2t,\,3-3t,\,4-4t)$$

$$= (2-3t,\,3-3t,\,4-3t), \qquad t\in[0,1].$$

$$\int_\gamma \nabla\times\vec{F}\,d\gamma = \int_0^1 \langle(\nabla\times\vec{F})(\gamma(t)),\gamma'(t)\rangle\,dt = \int_0^1 \langle(4.(3-3t)-1,\,0,\,1),(-3,-3,-3)\rangle$$

$$= \int_0^1 \langle(\underbrace{12-12t-1}_{11-12t},\,0,\,1),(-3,-3,-3)\rangle\,dt = \int_0^1 -33+36t-3\,dt$$

$$= \int_0^1 36t-36\,dt = (18t^2-36t)\Big|_0^1 = 18-36 = \boxed{-18}$$