Name: ANSWERS Lehman ID: ____________

By writing my name above, I acknowledge complying with the CUNY Academic Integrity Policy while completing this examination.

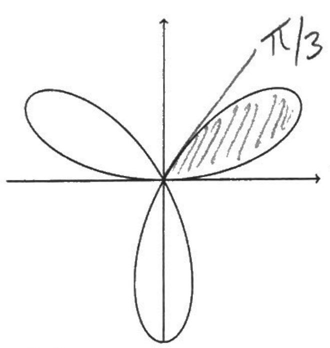
MAT 226 (Fall 2019)
Quiz 5

(5pts) Compute the triple integral $\int_0^1 \int_{-1}^1 \int_x^{2x} z \, dy \, dz \, dx$

$$\int_0^1\int_{-1}^1\int_x^{2x} z\,dy\,dz\,dx = \int_0^1\int_{-1}^1 zy\Big|_x^{2x}\,dz\,dx = \int_0^1\int_{-1}^1 z\cdot(2x-x)\,dz\,dx$$

$$= \int_0^1\int_{-1}^1 xz\,dz\,dx = \int_0^1 x\frac{z^2}{2}\Big|_{-1}^1\,dx = \int_0^1 0\,dx = \boxed{0}$$

(5pts) Find the area inside **one petal** of the 3-petal rose curve $r(\theta) = \sin(3\theta)$ below.



$$A = \int_0^{\pi/3}\int_0^{\sin(3\theta)} r\,dr\,d\theta$$

$$= \int_0^{\pi/3} \frac{r^2}{2}\Big|_0^{\sin 3\theta}\,d\theta$$

$$= \frac{1}{2}\int_0^{\pi/3} \sin^2(3\theta)\,d\theta$$

$$= \frac{1}{2}\int_0^{\pi/3} \frac{1-\cos(6\theta)}{2}\,d\theta$$

Note: $r(\theta)=0$ if and only if θ is one of the following angles: $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$.

$$= \frac{1}{4}\left(\int_0^{\pi/3} 1\,d\theta - \int_0^{\pi/3}\cos 6\theta\,d\theta\right) = \frac{1}{4}\left(\frac{\pi}{3} - 0\right) = \boxed{\frac{\pi}{12}}$$