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MAT 226 (Fall 2019)
Quiz 4

Find the **minimum and maximum** values that $f(x,y) = x - 2y$ assumes among the points (x,y) such that $x^2 + y^2 = 1$.

Target: $f(x,y) = x - 2y$

constraint: $g(x,y) = x^2 + y^2 = 1.$

Assuming $\lambda \neq 0$.
(If $\lambda = 0$, $\nabla f(x,y) = 0$ has no solutions.)

$\nabla f(x,y) = (1, -2) \qquad \nabla g(x,y) = (2x, 2y)$

$$\nabla f(x,y) = \lambda \nabla g(x,y) \iff \begin{cases} 1 = \lambda \cdot 2x \iff x = \dfrac{1}{2\lambda} \\ -2 = \lambda \cdot 2y \iff y = -\dfrac{1}{\lambda} \end{cases}$$

Plug back into constraint:

$$x^2 + y^2 = 1 \Rightarrow \left(\frac{1}{2\lambda}\right)^2 + \left(-\frac{1}{\lambda}\right)^2 = 1 \Rightarrow \frac{1}{4\lambda^2} + \frac{1}{\lambda^2} = 1$$

$$\Rightarrow \frac{1+4}{4\lambda^2} = 1 \Rightarrow \frac{5}{4} = \lambda^2 \Rightarrow \boxed{\lambda = \pm\frac{\sqrt{5}}{2}}$$

If $\lambda = \frac{\sqrt{5}}{2}$: $\quad x = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}, \quad y = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

If $\lambda = -\frac{\sqrt{5}}{2}$: $\quad x = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}, \quad y = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}.$

Minimum value is $f\left(-\frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}\right) = -\frac{\sqrt{5}}{5} - 2 \cdot \frac{2\sqrt{5}}{5} = \boxed{-\sqrt{5}}$

Maximum value is $f\left(\frac{\sqrt{5}}{5}, -\frac{2\sqrt{5}}{5}\right) = \frac{\sqrt{5}}{5} + \frac{4\sqrt{5}}{5} = \boxed{\sqrt{5}}$