

Name: ANSWERS Lehman ID: _____

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MAT 226 (Fall 2019)
Quiz 2

1. (5 pts) Compute the length $\|\gamma'(t)\|$ of the tangent vector to the curve

$$\gamma(t) = (t^2, \cos t + t \sin t, \sin t - t \cos t)$$

$$\begin{aligned}\gamma'(t) &= (2t, -\sin t + \sin t + t \cos t, \cos t - \cos t + t \sin t) \\ &= (2t, t \cos t, t \sin t)\end{aligned}$$

$$\begin{aligned}\|\gamma'(t)\| &= \sqrt{(2t)^2 + (t \cos t)^2 + (t \sin t)^2} \\ &= \sqrt{4t^2 + t^2 \cos^2 t + t^2 \sin^2 t} \\ &= \sqrt{4t^2 + t^2} = \sqrt{5t^2} = \boxed{t\sqrt{5}}\end{aligned}$$

2. (5 pts) Use the above computation to find the arclength of $\gamma(t)$ from $t = 0$ to $t = \sqrt{2}$.

$$\begin{aligned}\text{length} &= \int_0^{\sqrt{2}} \|\gamma'(t)\| dt \\ &= \int_0^{\sqrt{2}} t\sqrt{5} dt = \sqrt{5} \frac{t^2}{2} \Big|_0^{\sqrt{2}} \\ &= \frac{\sqrt{5}}{2} \cdot 2 = \boxed{\sqrt{5}}\end{aligned}$$