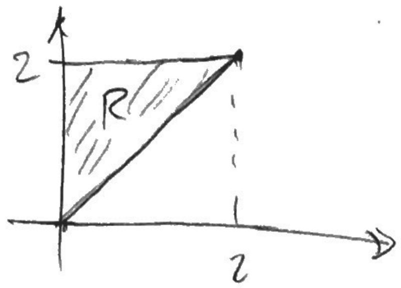
MAT 226

Solutions to HW9

a)



$$R = \{(x,y) \in \mathbb{R}^2 : 0 \le y \le 2,\ 0 \le x \le y\}$$

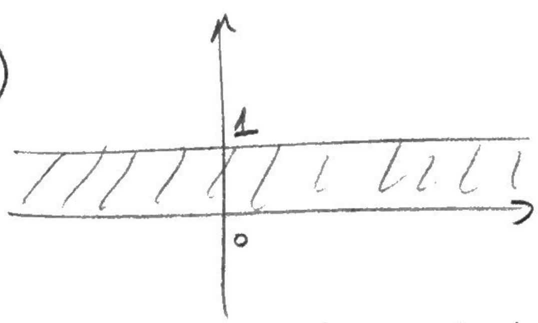
$$\iint_R \sqrt{4-y^2}\, dA = \int_0^2 \int_0^y \sqrt{4-y^2}\, dx\, dy =$$

$$= \int_0^2 x\sqrt{4-y^2}\Big|_0^y dy = \int_0^2 y\sqrt{4-y^2}\, dy = \int_4^0 u^{1/2} \frac{du}{-2} =$$

$u = 4-y^2$
$du = -2y\,dy$

$$= \frac{1}{2} \cdot \frac{u^{3/2}}{3/2}\Big|_0^4 = \frac{1}{3}\left(4^{3/2}\right) = \boxed{\frac{8}{3}}$$

b)

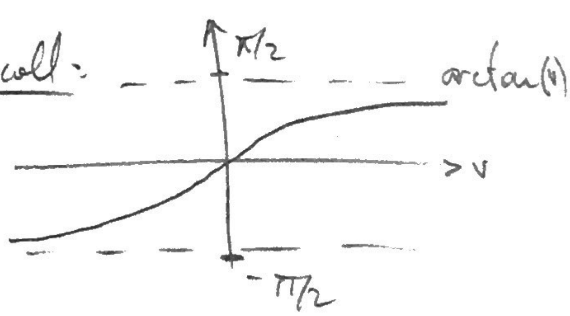


$$\iint_R \frac{y^2}{1+x^2}\, dA = \int_0^1 \int_{-\infty}^{+\infty} \frac{y^2\, dx}{1+x^2}\, dy$$

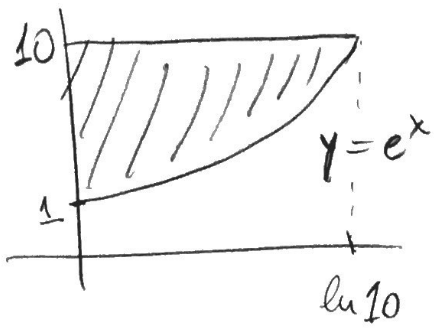
$$= \left(\int_0^1 y^2\, dy\right) \cdot \left(\int_{-\infty}^{+\infty} \frac{dx}{1+x^2}\right) = \frac{1}{3}\left(\lim_{b\to\infty} \arctan(x) - \lim_{a\to-\infty} \arctan(x)\right)$$

$$= \frac{1}{3}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = \boxed{\frac{\pi}{3}}$$

Recall:



c)



$R = \{(x,y) \in \mathbb{R}^2 : 0 \le x \le \ln 10,\ e^x \le y \le 10\}$

$$\iint_R \frac{1}{\ln y}\, dA = \int_0^{\ln 10} \underbrace{\int_{e^x}^{10} \frac{1}{\ln y}\, dy}_{\substack{\text{Can't integrate} \\ \text{(with elementary methods)}}} dx$$

Switch order: $y = e^x \iff \ln y = x$

$R = \{(x,y) \in \mathbb{R}^2 : 1 \le y \le 10,\ 0 \le x \le \ln y\}$

$$\iint_R \frac{1}{\ln y}\, dA = \int_1^{10} \int_0^{\ln y} \frac{1}{\ln y}\, dx\, dy = \int_1^{10} \left(\frac{x}{\ln y}\right)\Big|_0^{\ln y} dy$$

$$= \int_1^{10} 1\, dy = y\Big|_1^{10} = \boxed{9}$$