MAT 226 Solutions to HW 5

$$f(x,y) = x^4 - x^2y^2 + 3y + 8$$

a) $\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \boxed{\left(4x^3 - 2xy^2, -2x^2y + 3\right)}$

b) $\nabla f(1,1) = (2,1)$.

A vector $\vec{v}$ orthogonal to $\nabla f(1,1)$ is $\vec{v} = (-1,2)$.

Indeed: $\langle \nabla f(1,1), \vec{v} \rangle = \langle (2,1), (-1,2) \rangle = -2+2 = \underline{\underline{0}}$

Thus, a <u>unit</u> vector orthogonal to $\nabla f(1,1)$ is

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{5}}(-1,2) = \boxed{\left(-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)}$$

c) $\frac{\partial f}{\partial \vec{u}}(1,1) = \langle \nabla f(1,1), \vec{u} \rangle = \left\langle (2,1), \left(-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) \right\rangle$

$$= -\frac{2}{\sqrt{5}} + \frac{2}{\sqrt{5}} = \underline{\underline{0}}$$

The derivative $\frac{\partial f}{\partial \vec{u}}$ vanishes because $\vec{u}$ is <u>orthogonal</u> to $\nabla f(1,1)$, hence <u>tangent</u> to a <u>levelset</u> of $f(x,y)$. Thus, $f(x,y)$ remains (infinitesimally) <u>unchanged</u> when moving in this direction, that is, the directional derivative $\frac{\partial f}{\partial \vec{u}}$ is zero.