MAT 226

# Solution to HW 1

1. $\vec{v} = (1,2,3)$, $\vec{w} = (-1,0,1)$

a) $\vec{v} + \vec{w} = \boxed{(0,2,4)}$

b) $2\vec{v} - 3\vec{w} = (2,4,6) + (3,0,-3) = \boxed{(5,4,3)}$

c) $\langle \vec{v}, \vec{w} \rangle = 1.(-1) + 2.0 + 3.1 = -1 + 3 = \boxed{2}$

d) $\langle \vec{v}+\vec{w}, \vec{v} - 2\vec{w} \rangle = \|\vec{v}\|^2 - 2\langle \vec{v}, \vec{w} \rangle + \langle \vec{w}, \vec{v} \rangle - 2\|\vec{w}\|^2$

$= \|\vec{v}\|^2 - \langle \vec{v}, \vec{w} \rangle - 2\|\vec{w}\|^2$

$= (1+4+9) - 2 - 2(2)$

$= 14 - 2 - 4$

$= \boxed{8}$

2. $\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 0 & 1 \end{vmatrix} = (2, -1-3, 2)$

$= (2, -4, 2)$

$\|\vec{v} \times \vec{w}\|^2 = 4 + 16 + 4 = 24 \Rightarrow \|\vec{v} \times \vec{w}\| = \sqrt{24} = 2\sqrt{6}$

$\dfrac{\vec{v} \times \vec{w}}{\|\vec{v} \times \vec{w}\|} = \dfrac{1}{2\sqrt{6}} (2, -4, 2) = \boxed{\left( \dfrac{1}{\sqrt{6}}, -\dfrac{2}{\sqrt{6}}, \dfrac{1}{\sqrt{6}} \right)}$