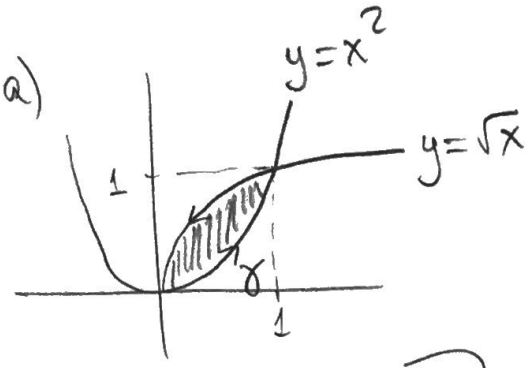
MAT 226

Solutions to HW 13

a)

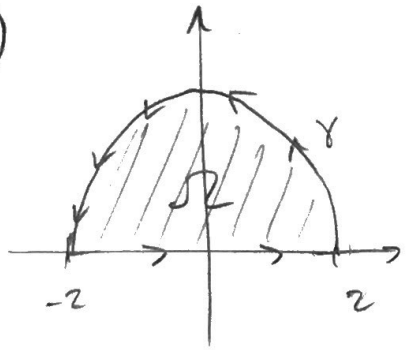


$$\int_\gamma (y+e^{\sqrt{x}})\,dx + (2x+\cos(y^2))\,dy$$

Green's Thm $\Rightarrow = \iint_\Omega (2-1)\,dA = \int_0^1 \int_{x^2}^{\sqrt{x}} dy\,dx$

$$= \int_0^1 (\sqrt{x}-x^2)\,dx = \left(\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right)\Big|_0^1 = \frac{2}{3} - \frac{1}{3} = \boxed{\frac{1}{3}}$$

b)

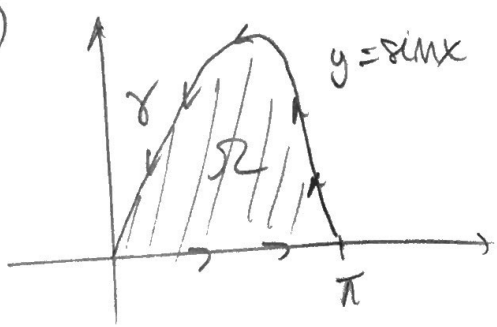


$$\int_\gamma xy\,dx + 2x^2\,dy \overset{\text{Green's thm}}{=} \iint_\Omega 4x - x\,dA$$

$$= 3\iint_\Omega x\,dA = 3\int_0^\pi \int_0^2 r\cos\theta \cdot r\,dr\,d\theta$$

$$= 3\underbrace{\int_0^\pi \cos\theta\,d\theta}_{=0} \cdot \int_0^2 r^2\,dr = \boxed{0}$$

c)



$$\int_\gamma (xy+e^{x^2})\,dx + (x^2-\ln(1+y))\,dy \overset{\text{Green's Thm}}{=}$$

$$= \iint_\Omega 2x - x\,dA = \iint_\Omega x\,dA = \int_0^\pi \int_0^{\sin x} x\,dy\,dx$$

$$= \int_0^\pi x\sin x\,dx = (-x\cos x + \sin x)\Big|_0^\pi = \boxed{\pi}$$