MAT 226

Solutions to HW 12

1) $\gamma(t) = t(3,4) + (1-t)(1,1) = (3t, 4t) + (1-t, 1-t)$

$= (2t+1, 3t+1) \quad , \quad t \in [0,1].$

$$\int_\gamma \vec{F}\, d\gamma = \int_0^1 \langle \vec{F}(\gamma(t)), \gamma'(t) \rangle \, dt$$

$$= \int_0^1 \Big\langle \big((2t+1)(3t+1) - 1, (2t+1)^2 + 3(3t+1) \big), (2,3) \Big\rangle \, dt$$

$$= \int_0^1 \Big\langle \big(6t^2 + 5t, \underbrace{4t^2 + 4t + 1 + 9t + 3}_{4t^2 + 13t + 4} \big), (2,3) \Big\rangle \, dt$$

$$= \int_0^1 12t^2 + 10t + 12t^2 + 39t + 12 \, dt$$

$$= \int_0^1 24t^2 + 49t + 12\, dt = \left(8t^3 + 49\frac{t^2}{2} + 12t \right) \Big|_0^1$$

$$= 8 + \frac{49}{2} + 12 = \frac{40+49}{2} = \boxed{\frac{89}{2}}$$

2) $$\nabla \times \vec{F}(x,y,z) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -z & y+1 & x \end{vmatrix}$$

$$\nabla \times \vec{F}(x,y,z) = (0, -1-1, 0) = (0, -2, 0).$$

$$\int_\gamma \nabla\times F\, d\gamma = \int_0^\pi \Big\langle \underbrace{(0,-2,0)}_{(\nabla\times F)(\gamma(t))}, \underbrace{(\cos t - t\sin t,\ \sin t + t\cos t,\ 1)}_{\gamma'(t)} \Big\rangle\, dt$$

$$= \int_0^\pi -2\sin t - 2t\cos t\, dt$$

$$= +2\cos t \Big|_0^\pi - 2\Big(t\sin t + \cos t\Big)\Big|_0^\pi$$

$$= 2(\cos\pi - \cos 0) - 2(\pi\sin\pi + \cos\pi - \cos 0)$$

$$= 2(-1-1) - 2(-1-1) = -4 + 4 = 0$$