

Practice Problems for the Midterm Exam

1. Compute the following integrals:

a) $\int x \sin(x) \, dx$

b) $\int x e^{-x} \, dx$

c) $\int x^2 \ln(x) \, dx$

d) $\int x^2 e^x \, dx$

e) $\int x \sqrt{9 - x^2} \, dx$

f) $\int \sqrt{9 - x^2} \, dx$

2. Compute the following improper integrals if it converges, or show it diverges:

a) $\int_0^{+\infty} x \sin(x) \, dx$

b) $\int_0^{+\infty} x e^{-x} \, dx$

c) $\int_1^{+\infty} \frac{3}{x^2} \, dx$

d) $\int_0^1 \frac{3}{x^2} \, dx$

e) $\int_{-\infty}^{+\infty} \frac{e^x}{e^x + 1} \, dx$

f) $\int_{-\infty}^{+\infty} \frac{e^{-2x}}{4\pi} \, dx$

3. Use a trigonometric substitution to compute the following integrals:

a) $\int \frac{3}{\sqrt{9 - x^2}} \, dx$

b) $\int \sqrt{1 + t^2} \, dt$

c) $\int \sqrt{1 - 4x^2} \, dx$

4. Solve the following initial value problems:

a) $\frac{dy}{dx} = \sin^5 x \cos^6 x, \quad y(0) = 0$

b) $\frac{dy}{dx} = x \sin^2 x, \quad y(0) = 0$

c) $\frac{dy}{dx} = \sqrt{4 - x^2}, \quad y(0) = 1$

5. Let R be the region bounded by the graph of $y = x^3$ and the lines $y = 0$, $x = 0$, and $x = 2$. Sketch a picture of the region R and use it to write down (but do not evaluate) integrals that compute the volume of the solids obtained by revolving R about the following lines:

a) $y = 0$

b) $y = -2$

c) $y = 15$

d) $x = 0$

e) $x = -1$

f) $x = 4$

6. What is the volume of the solid obtained by revolving the region bounded by the line $y = 2x$ and the parabola $y = 2 - x^2$ about the line $x = 1$?

7. Find the volume of the solid obtained by revolving the region bounded between the x -axis and the curve $y = \frac{e^x + e^{-x}}{2}$, $-2 \leq x \leq 2$, about the x -axis.

8. A dosage of 300mg of painkiller is given to a patient at 11:00am each day. Suppose 20% of the drug remains in the body after one full day period (11:00am next day).

a) What is the amount of painkiller in the body 1 day after the treatment started, before the next dose is given at 11:00am?

b) What is the amount of painkiller in the body 2 days after the treatment started, before the next dose is given at 11:00am?

c) What is the amount of painkiller in the body 3 days after the treatment started, before the next dose is given at 11:00am?

d) Use a geometric series to estimate the amount of painkiller in the body after a very long time of treatment before a new dose is given.

9. Suppose that a basketball rebounds $2/3$ of its previous height after each bounce. If this basketball is dropped from a height of 3m, how far does it travel up and down until it stops moving?
10. Determine whether each of the following series converges or diverges. Carefully specify the convergence tests that are being used.

a) $\sum_{n=1}^{+\infty} n^2$

b) $\sum_{n=1}^{+\infty} \frac{2}{5^n}$

c) $\sum_{n=1}^{+\infty} \frac{n^2 - 1}{n^2 + 1}$

d) $\sum_{n=1}^{+\infty} \frac{1}{n^2 + 4}$

e) $\sum_{n=1}^{+\infty} \frac{n}{n^2 + 4}$

f) $\sum_{n=1}^{+\infty} \frac{1}{\sqrt{n} + 1}$

g) $\sum_{n=1}^{+\infty} \frac{n}{e^n}$