Solutions to HW 6

a) $$\int_0^{+\infty} x^2 e^{-x}\,dx = \lim_{b\to\infty} \int_0^b x^2 e^{-x}\,dx = \dots$$

Integration by parts (twice):

$$\int x^2 e^{-x}\,dx = -x^2 e^{-x} + \int 2x e^{-x}\,dx$$

$$= -x^2 e^{-x} - 2x e^{-x} + \int 2 e^{-x}\,dx$$

$$= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C$$

$$= (-x^2 - 2x - 2) e^{-x} + C = -(x^2 + 2x + 2) e^{-x} + C$$

so, plug it in..

$$\dots = \lim_{b\to\infty} -(x^2+2x+2)e^{-x}\Big|_0^b = \lim_{b\to\infty} -(b^2+2b+2)e^{-b} + 2e^0$$

$$= \lim_{b\to\infty} \underbrace{-b^2 e^{-b}}_{\downarrow\, 0} \underbrace{- 2be^{-b}}_{\downarrow\, 0} \underbrace{- 2e^{-b}}_{\downarrow\, 0} + 2 = 2$$

by L'Hospital

b) $$\int_{-\infty}^{+\infty} \frac{t^2}{t^2+1}\,dt \overset{\text{Hint}}{=} \int_{-\infty}^{+\infty} 1 - \frac{1}{t^2+1}\,dt$$

$$= \lim_{a\to-\infty} \int_a^0 1 - \frac{1}{t^2+1}\,dt + \lim_{b\to\infty} \int_0^b 1 - \frac{1}{1+t^2}\,dt$$

$$= \lim_{a \to -\infty} \left(t - \arctan t \right) \Big|_a^0 + \lim_{b \to \infty} \left(t - \arctan t \right) \Big|_0^b$$

$$= \underbrace{\lim_{a \to -\infty} -a + \arctan a}_{+\infty} + \underbrace{\lim_{b \to \infty} b - \arctan b}_{+\infty}$$

$$= +\infty$$ (diverge).