

Solutions to HW2

$$a) \int e^{-t} \cos t \, dt \stackrel{\text{Int. by Parts } \left(\begin{array}{l} u = e^{-t} \\ du = -e^{-t} dt \end{array} \quad \begin{array}{l} v = \sin t \\ dv = \cos t \, dt \end{array} \right)}{=} e^{-t} \sin t + \int e^{-t} \sin t \, dt$$

$$\stackrel{\text{Int. by Parts } \left(\begin{array}{l} u = e^{-t} \\ du = -e^{-t} dt \end{array} \quad \begin{array}{l} v = -\cos t \\ dv = \sin t \, dt \end{array} \right)}{=} e^{-t} \sin t - e^{-t} \cos t - \underbrace{\int e^{-t} \cos t \, dt}_I$$

$$\Rightarrow 2I = 2 \int e^{-t} \cos t \, dt = e^{-t} (\sin t - \cos t) + C$$

$$\Rightarrow I = \int e^{-t} \cos t \, dt = \frac{e^{-t}}{2} (\sin t - \cos t) + C$$

$$b) \int \sin^4 x \cos^3 x \, dx = \int \sin^4 x \cdot \cos^2 x \cos x \, dx$$

$$= \int \sin^4 x (1 - \sin^2 x) \cos x \, dx$$

$$\left. \begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array} \right\} \Rightarrow \int u^4 (1 - u^2) \, du$$

$$= \int u^4 - u^6 \, du$$

$$= \frac{u^5}{5} - \frac{u^7}{7} + C$$

$$= \frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C$$