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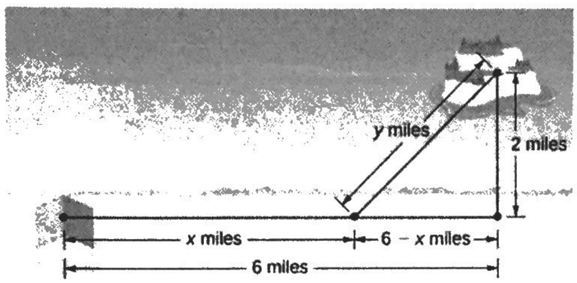
# MAT175 (Spring 2019)
## Quiz 5

1. An island is 2 miles north of its closest point along a straight shoreline. A visitor is on the shore, 6 miles west of that point, and wants to go to the island. Suppose the visitor runs at a rate of 9 mph and swims at a rate of 3mph. How far should the visitor run before swimming to minimize the time it takes to reach the island?




$$\left(\begin{matrix}\text{time}\\\text{running}\end{matrix}\right) = \frac{\overbrace{x}^{\text{distance running}}}{\underbrace{9}_{\text{running speed}}}$$

$$\left(\begin{matrix}\text{time}\\\text{swimming}\end{matrix}\right) = \frac{\overbrace{y}^{\text{distance swimming}}}{\underbrace{3}_{\text{swimming speed}}}$$

$$\text{total time} = \frac{x}{9} + \frac{y}{3}$$

$$\text{constraint:} \quad y^2 = (6-x)^2 + 2^2 = (x-6)^2 + 4$$

$$\text{total time:} \quad T(x) = \frac{x}{9} + \frac{1}{3}\sqrt{(x-6)^2+4}\,, \quad x \in [0,6]$$

$$T'(x) = \frac{1}{9} + \frac{1}{3}\,\frac{2(x-6)}{2\sqrt{(x-6)^2+4}} = \frac{1}{9} + \frac{x-6}{3\sqrt{(x-6)^2+4}}$$

$$\text{So } T'(x) = 0 \iff x-6 = -\frac{\sqrt{(x-6)^2+4}}{3}$$

$$\iff (x-6)^2 = \frac{1}{9}\left((x-6)^2+4\right)$$

$$\iff \frac{8}{9}(x-6)^2 = \frac{4}{9} \iff (x-6)^2 = \frac{1}{2} \iff x-6 = \pm\frac{1}{\sqrt{2}}$$

$$\iff x = 6 \pm \frac{\sqrt{2}}{2}.$$

$\longrightarrow$

Only critical point in $[0,6]$: $x = 6 - \frac{\sqrt{2}}{2}$

$T\left(6-\frac{\sqrt{2}}{2}\right) = \frac{2}{3} + \frac{4\sqrt{2}}{9} \cong 1.295$ $\leftarrow$ Global minimum of $T(x)$ on the interval $[0,6]$.

Endpoints

swim only:

$T(0) = \frac{2\sqrt{10}}{3} \cong 2.108$

run max distance:

$T(6) = \frac{4}{3} \cong 1.333$

A: The global min of $T: [0,6] \to \mathbb{R}$ is achieved at $x = 6 - \frac{\sqrt{2}}{2}$, which means that the fastest way to get to the island is to run $\left(6-\frac{\sqrt{2}}{2}\right)$ mi and swim the rest of the distance.