Name: ANSWERS Lehman ID: _____

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MAT175 (Spring 2019)
Quiz 1

1. **(5 pts)** Compute $\lim_{\theta \to 0} \dfrac{e^{3\theta}\sin(7\theta)\cos(2\theta)}{5\theta}$

$$\lim_{\theta\to 0} \frac{e^{3\theta}\sin(7\theta)\cos(2\theta)}{5\theta} = \lim_{\theta\to 0} \left(e^{3\theta}\right)\left(\frac{\sin(7\theta)}{7\theta}\right)\cdot\left(\frac{7}{5}\cos 2\theta\right)$$

$$= \underbrace{\left(\lim_{\theta\to 0} e^{3\theta}\right)}_{1}\underbrace{\left(\lim_{\theta\to 0}\frac{\sin(7\theta)}{7\theta}\right)}_{1}\underbrace{\left(\lim_{\theta\to 0}\frac{7}{5}\cos 2\theta\right)}_{7/5}$$

$$= \boxed{\frac{7}{5}}$$

2. **(5 pts)** For what value of a is the function $f(x)$ continuous everywhere?

$$f(x) = \begin{cases} \dfrac{x^2-4}{4x-8}, & \text{if } x < 2 \\ ax^3, & \text{if } x \geq 2 \end{cases}$$

In order for $f(x)$ to be continuous at $x=2$, need the lateral limits $x \to 2-$ and $x \to 2+$ to agree;

$$\lim_{x\to 2-} f(x) = \lim_{x\to 2-}\frac{x^2-4}{4x-8} = \lim_{x\to 2-}\frac{(x+2)(x-2)}{4(x-2)} =$$

$$= \lim_{x\to 2-}\frac{x+2}{4} = \boxed{1}$$

$$\lim_{x\to 2+} f(x) = \lim_{x\to 2+} ax^3 = \boxed{8a}$$

So we need $1 = 8a$

$$\boxed{a = \frac{1}{8}}$$

If $\boxed{a = \frac{1}{8}}$, then $f(2) = \lim_{x\to 2} f(x) = 1$ is continuous at $x=2$, and it is clearly continuous at every other $x \neq 2$.